GRAĐEVINSKA FIZIKA - RAČUNSKE VEŽBE - III NEDELJA

1. Sferni zid, koji je deo termičkog omotača zgrade, sastoji se od pet slojeva. Unutrašnji poluprečnik posmatrane strukture je 3,2m. Debljine i toplotne provodnosti pojedinih slojeva su date u donjoj tabeli. Prvi sloj u tabeli se nalazi sa unutrašnje strane zida, a poslednji sa spoljašnje. Koeficijenti prelaza toplote sa unutrašnje i spoljašnje strane zida su $7{,}69\ \frac{W}{K \cdot m^2}$. Spoljašnja temperatura je 3°C, a temperatura unutar prostorije je 26°C. Ako su termalni kontakti između slojeva idealni, odrediti:

a) Toplotni fluks

b) Ukupnu termalnu otpornost prolaza toplote posmatranog zida

c) Temperaturu zida sa spoljašnje i unutrašnje strane

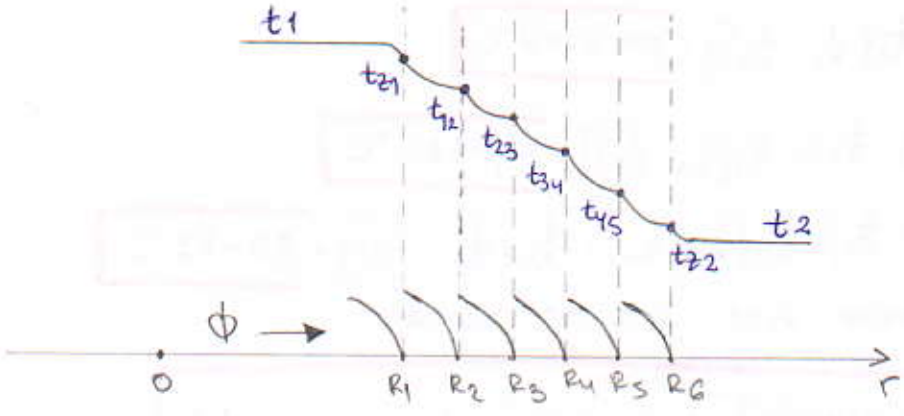
d) Temperature na spojevima materijala

e) Temperatursko polje unutar kamene vune i giter bloka

Br.	Materijal	Debljina d[cm]	$\lambda \left[\frac{W}{m \cdot K}\right]$
1	Gips	2	0,5
2	Malter (unutra)	3	1,40
3	Giter blok	30	0,61
4	Kamena vuna	8	0,034
5	Malter (spolja)	2	1,40

$t_1 = 26°C \quad t_2 = 3°C$

$\alpha_1 = 7{,}69\ \frac{W}{m^2 K} \quad \alpha_2 = 7{,}69\ \frac{W}{m^2 K}$



$R_1 = 3{,}2m$

$R_2 = R_1 + d_1 = 3{,}2 + 0{,}02 = 3{,}22m$

$R_3 = R_2 + d_2 = 3{,}25m$

$R_4 = R_3 + d_3 = 3{,}55m$

$R_5 = R_4 + d_4 = 3{,}63m$

$R_6 = R_5 + d_5 = 3{,}65m$

a) Toplotni fluks

$$\Phi = \frac{t_1 - t_2}{R_T} = \frac{t_1 - t_2}{\frac{1}{2\pi}\left[\frac{1}{\alpha_1 R_1^2} + \sum_{i=1}^{5} \frac{1}{\lambda_i}\left(\frac{1}{R_i} - \frac{1}{R_{i+1}}\right) + \frac{1}{\alpha_2 R_6^2}\right]}$$

$$\Phi = \frac{26 - 3}{\frac{1}{2\pi}\left[\frac{1}{7{,}69 \cdot (3{,}2)^2} + \frac{1}{0{,}5}\left(\frac{1}{3{,}2} - \frac{1}{3{,}22}\right) + \frac{1}{1{,}4}\left(\frac{1}{3{,}22} - \frac{1}{3{,}25}\right) + \frac{1}{0{,}61}\left(\frac{1}{3{,}25} - \frac{1}{3{,}55}\right) + \frac{1}{0{,}034}\left(\frac{1}{3{,}55} - \frac{1}{3{,}63}\right) + \frac{1}{1{,}4}\left(\frac{1}{3{,}63} - \frac{1}{3{,}65}\right) + \frac{1}{7{,}69 \cdot (3{,}65)^2}\right]}$$

$$\Phi = \frac{23}{90405034\,20829} = 567{,}42196 \ W$$

b) Termalna otpornost prolaza toplote

$$\Phi = \frac{\Delta t}{R_T} \Rightarrow R_T = \Phi \cdot \Delta t \qquad R_T = 90405 \ \frac{K}{W}$$

c) Temperature zida sa spoljašnje i unutrašnje strane

$$t_{z1} = t_1 - \frac{\Phi}{2\pi \alpha_1 R_1^2} = 24{,}853 \ ^\circ C$$

$$t_{z2} = t_2 + \frac{\Phi}{2\pi \alpha_2 R_6^2} = 3{,}8815 \ ^\circ C$$

d) Temperature na spojevima materijala

$$t_{12} = t_1 - \frac{\Phi}{2\pi}\left[\frac{1}{\alpha_1 R_1^2} + \frac{1}{\lambda_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right] = 24{,}5026 \ ^\circ C$$

$$t_{23} = t_1 - \frac{\Phi}{2\pi}\left[\frac{1}{\alpha_1 R_1^2} + \frac{1}{\lambda_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{1}{\lambda_2}\left(\frac{1}{R_2} - \frac{1}{R_3}\right)\right] = 24{,}3177 \ ^\circ C$$

$$t_{34} = t_1 - \frac{\Phi}{2\pi}\left[\frac{1}{\alpha_1 R_1^2} + \frac{1}{\lambda_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{1}{\lambda_2}\left(\frac{1}{R_2} - \frac{1}{R_3}\right) + \frac{1}{\lambda_3}\left(\frac{1}{R_3} - \frac{1}{R_4}\right)\right] = 20{,}4682 \ ^\circ C$$

$$t_{45} = t_1 - \frac{\Phi}{2\pi}\left[\frac{1}{\alpha_1 R_1^2} + \frac{1}{\lambda_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{1}{\lambda_2}\left(\frac{1}{R_2} - \frac{1}{R_3}\right) + \frac{1}{\lambda_3}\left(\frac{1}{R_3} - \frac{1}{R_4}\right) + \frac{1}{\lambda_4}\left(\frac{1}{R_4} - \frac{1}{R_5}\right)\right] = 3{,}9788 \ ^\circ C$$

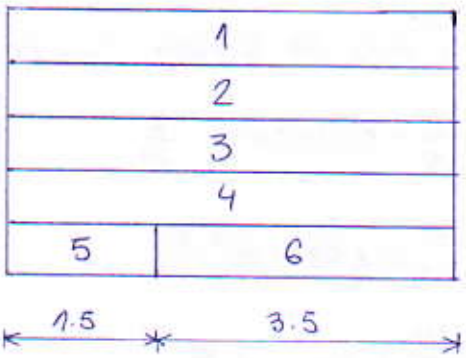
e) Temperatursko polje unutar kamene vune i giter bloka

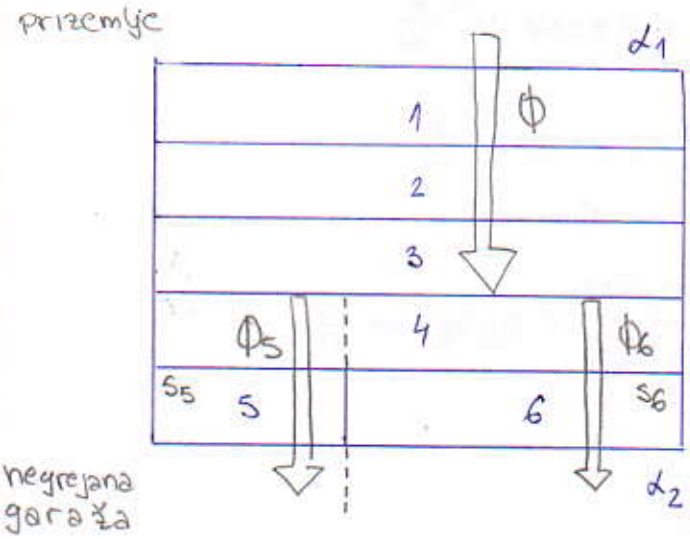
$$t_3(r) = t_{23} - \frac{\Phi}{2\pi} \cdot \frac{1}{\lambda_3}\left(\frac{1}{R_3} - \frac{1}{r}\right) = 24{,}3177\,[^\circ C] - 148{,}045\,[^\circ C \cdot m]\left(0{,}30769 - \frac{1}{r}\right)$$

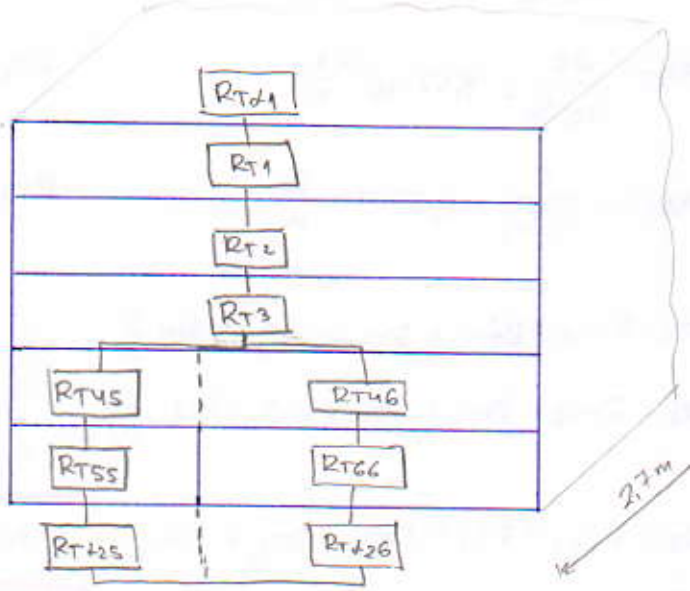
$$t_4(r) = t_{34} - \frac{\Phi}{2\pi}\frac{1}{\lambda_4}\left(\frac{1}{R_4} - \frac{1}{r}\right) = 20{,}4682\,[^\circ C] - 2656{,}118\,[^\circ C \cdot m]\left(0{,}28169 - \frac{1}{r}\right)$$

(2.) Izračunati ekvivalentni termički otpor prolaza toplote za ravan višeslojni zid i R-vrednost zida dužine 5m i širine 2,7m, čije su dimenzije i poprečni presek prikazani na slici. Zid je horizontalna međuspratna konstrukcija ka negrejanoj garaži i deo je termičkog omotača zgrade. Koeficijenti prelaza toplote sa spoljašnje i unutrašnje strane zida su jednaki i iznose 5,88 $\frac{W}{m^2 K}$. Koeficijenti toplotne provodnosti za pojedine materijale su dati u tabeli. Prvi sloj u tabeli se nalazi sa gornje, a poslednji sa donje strane konstrukcije.

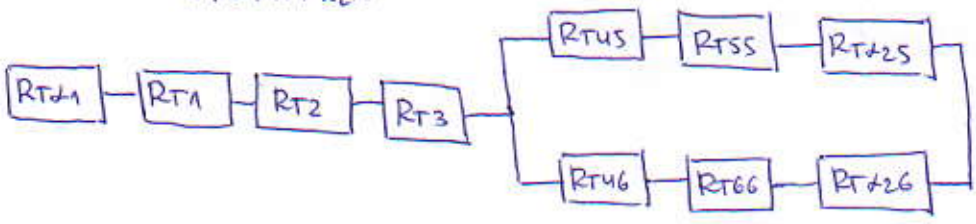
Br.	Materijal	Debljina d[cm]	$\lambda \left[\frac{W}{m \cdot K}\right]$
1	Keramičke pločice	1	1,28
2	Cementni estrih	3	1,40
3	Giter blok	10	0,61
4	Stirodur	3	0,035
5	Malter	2	1,40
6	Gips	2	0,70





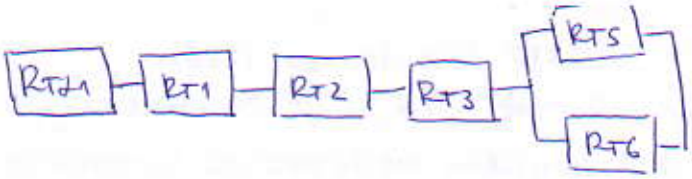


⇒ Šema otpornika:

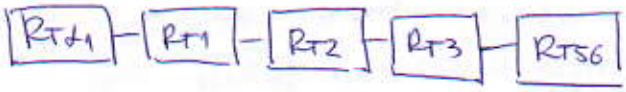


$R_{T5} = R_{T45} + R_{T55} + R_{T\alpha 25}$

$R_{T6} = R_{T46} + R_{T66} + R_{T\alpha 26}$



$$\frac{1}{R_{T56}} = \frac{1}{R_{T5}} + \frac{1}{R_{T6}} \Rightarrow R_{T56} = \frac{R_{T5} \cdot R_{T6}}{R_{T5} + R_{T6}}$$



$$R_T = R_{Td1} + R_{T1} + R_{T2} + R_{T3} + R_{T56}$$

- ukupna površina zida: $S = 5 \cdot 2{,}7 = 13{,}5\,m^2$
- površina zida sa malterom: $S_5 = 1{,}5 \cdot 2{,}7 = 4{,}05\,m^2$
- površina zida sa gipsom: $S_6 = 3{,}5 \cdot 2{,}7 = 9{,}45\,m^2$

$$R_{Td1} = \frac{1}{\lambda_1 \cdot S} = 0{,}0125976 \frac{k}{W} \qquad R_{T1} = \frac{d_1}{\lambda_1 \cdot S} = 5{,}787 \cdot 10^{-4} \frac{k}{W}$$

$$R_{T2} = \frac{d_2}{\lambda_2 \cdot S} = 1{,}587 \cdot 10^{-3} \frac{k}{W} \qquad R_{T3} = \frac{d_3}{\lambda_3 \cdot S} = 0{,}01214 \frac{k}{W}$$

$$R_{T45} = \frac{d_4}{\lambda_4 \cdot S_5} = 0{,}21164 \frac{k}{W} \qquad R_{T46} = \frac{d_4}{\lambda_4 \cdot S_6} = 0{,}0907 \frac{k}{W}$$

$$R_{T55} = \frac{d_5}{\lambda_5 \cdot S_5} = 3{,}52 \cdot 10^{-3} \frac{k}{W} \qquad R_{T66} = \frac{d_6}{\lambda_6 \cdot S_6} = 3{,}0234 \cdot 10^{-3} \frac{k}{W}$$

$$R_{Td25} = \frac{1}{\alpha_2 \cdot S_5} = 0{,}04199 \frac{k}{W} \qquad R_{Td26} = \frac{1}{\alpha_2 \cdot S_6} = 0{,}017997 \frac{k}{W}$$

$$R_{T5} = R_{T45} + R_{T55} + R_{Td25} = 0{,}25715 \frac{k}{W}$$
$$R_{T6} = R_{T46} + R_{T66} + R_{Td26} = 0{,}11192 \frac{k}{W}$$
$$\Rightarrow R_{T56} = \frac{R_{T5} \cdot R_{T6}}{R_{T5} + R_{T6}} = 0{,}076743 \frac{k}{W}$$

$$R_T = R_{Td1} + R_{T1} + R_{T2} + R_{T3} + R_{T56} = 0{,}10364 \frac{k}{W}$$

→ R-vrednost zida: $R = R_T \cdot S$

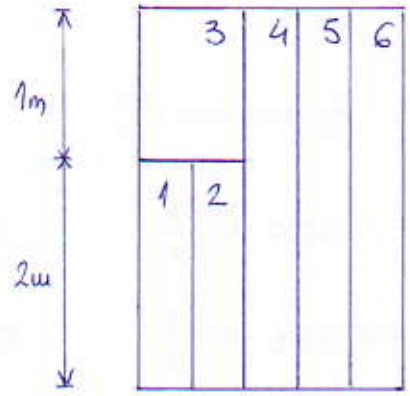
$$R_5 = R_{T5} \cdot S_5 = 1{,}04015 \frac{k \cdot m^2}{W} \qquad R_6 = R_{T6} \cdot S_6 = 1{,}10754 \frac{k \cdot m^2}{W}$$

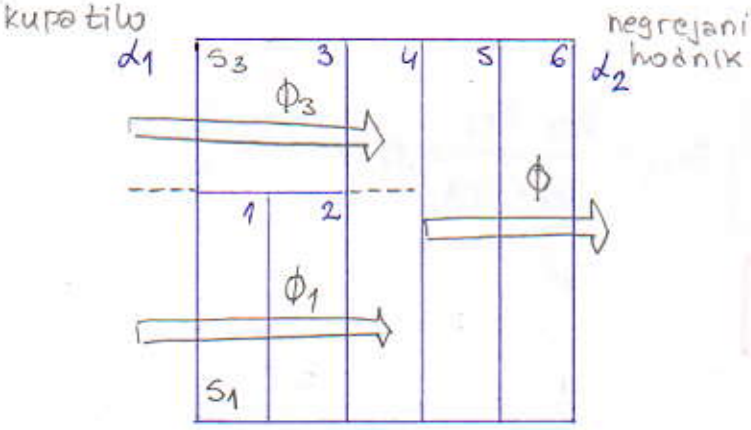
$$R_{56} = \frac{R_5 \cdot R_6}{R_5 + R_6} = 0{,}5364 \frac{k m^2}{W}$$

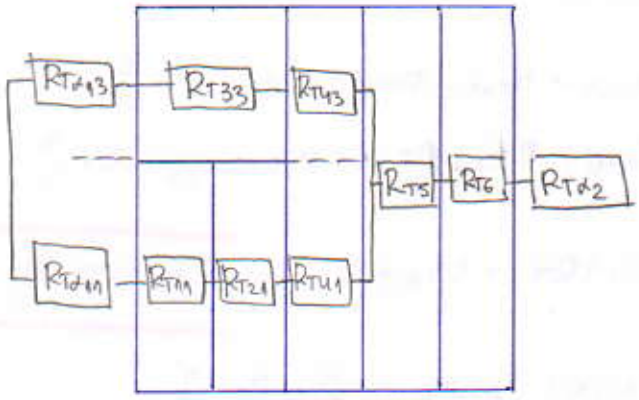
$$R = (R_{Td1} + R_{T1} + R_{T2} + R_{T3}) \cdot S + R_{56} = 0{,}89994 \frac{k \cdot m^2}{W}$$

③ Izračunati ekvivalentni termički otpor za prolaz toplote i R vrednost zida za ravan višeslojni zid dužine 3m i visine 3,5m čije su dimenzije i poprečni presek prikazani na slici. Zid razdvaja kupatilo od negrejanog hodnika i deo je termičkog omotača zgrade. Koeficijenti prelaza toplote sa spoljašnje i unutrašnje strane zida su jednaki i iznose 7,69 $\frac{W}{K \cdot m^2}$. Koeficijenti toplotne provodnosti za pojedine materijale su dati u tabeli. Prvi sloj u tabeli se nalazi sa unutrašnje strane zida, a poslednji sa spoljašnje.

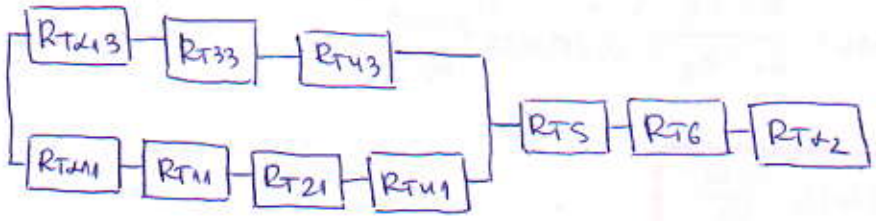
Br.	Materijal	Debljina d[cm]	$\lambda \left[\frac{W}{m \cdot K}\right]$
1	Keramičke pločice	1	1,28
2	Cementni estrih	1,5	1,4
3	Krečnogipsani malter	2,5	0,7
4	Armirani beton	12	2,33
5	Kamena vuna	4	0,04
6	Malter	1	1,4





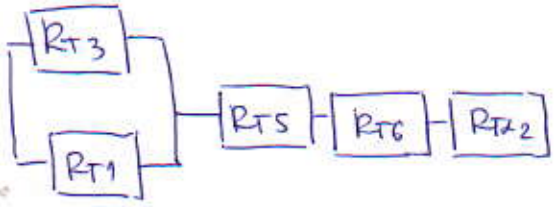


→ Šema otpornika:



$$R_{T3} = R_{T\alpha 13} + R_{T33} + R_{T43}$$

$$R_{T1} = R_{T\alpha 11} + R_{T11} + R_{T21} + R_{T41}$$



$$\frac{1}{R_{T13}} = \frac{1}{R_{T1}} + \frac{1}{R_{T3}} \Rightarrow R_{T13} = \frac{R_{T1} \cdot R_{T3}}{R_{T1} + R_{T3}}$$

$$R_T = R_{T13} + R_{T5} + R_{T6} + R_{T\alpha 2}$$

→ ukupna površina zida : $S = 3 \cdot 3{,}5 = 10{,}5\,m^2$

→ površina zida sa malterom: $S_3 = 1 \cdot 3{,}5 = 3{,}5\,m^2$

→ površina zida sa keramičkim pločicama: $S_1 = 2 \cdot 3{,}5 = 7\,m^2$

$$R_{T\alpha_2} = \frac{1}{\alpha_2 \cdot S} = 0{,}012385\ \frac{K}{W} \qquad R_{T6} = \frac{d_6}{\lambda_6 \cdot S} = 6{,}8027 \cdot 10^{-4}\ \frac{K}{W}$$

$$R_{T5} = \frac{d_5}{\lambda_5 \cdot S} = 0{,}09524\ \frac{K}{W}$$

$$R_{T\alpha 11} = \frac{1}{\alpha_1 \cdot S_1} = 0{,}0185774\ \frac{K}{W} \qquad R_{T\alpha 13} = \frac{1}{\alpha_1 \cdot S_3} = 0{,}037154\ \frac{K}{W}$$

$$R_{T11} = \frac{d_1}{\lambda_1 \cdot S_1} = 1{,}11607 \cdot 10^{-3}\ \frac{K}{W} \qquad R_{T33} = \frac{d_3}{\lambda_3 \cdot S_3} = 0{,}0102\ \frac{K}{W}$$

$$R_{T21} = \frac{d_2}{\lambda_2 \cdot S_1} = 1{,}5306 \cdot 10^{-3}\ \frac{K}{W} \qquad R_{T43} = \frac{d_4}{\lambda_4 \cdot S_3} = 0{,}0147\ \frac{K}{W}$$

$$R_{T41} = \frac{d_4}{\lambda_4 \cdot S_1} = 7{,}3574 \cdot 10^{-3}\ \frac{K}{W}$$

$$R_{T3} = R_{T\alpha 13} + R_{T33} + R_{T43} = 0{,}06205\ \frac{K}{W}$$

$$R_{T1} = R_{T\alpha 11} + R_{T11} + R_{T21} + R_{T41} = 0{,}02858\ \frac{K}{W}$$

$$R_{T13} = \frac{R_{T1} \cdot R_{T3}}{R_{T1} + R_{T3}} = 0{,}019567\ \frac{K}{W}$$

$$R_T = R_{T13} + R_{T5} + R_{T6} + R_{T\alpha 2} = 0{,}12787\ \frac{K}{W}$$

- R-vrednost zida $R = R_T \cdot S$

$$R_3 = R_{T3} \cdot S_3 = 0{,}217175\ \frac{K \cdot m^2}{W}$$

$$R_1 = R_{T1} \cdot S_1 = 0{,}20006\ \frac{K \cdot m^2}{W}$$

$$R_{13} = \frac{R_1 \cdot R_3}{R_1 + R_3} = 0{,}10413\ \frac{K \cdot m^2}{W}$$

$$R = R_{13} + (R_{T5} + R_{T6} + R_{T\alpha 2}) \cdot S = 1{,}2413\ \frac{K \cdot m^2}{W}$$

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