GRAĐEVINSKA FIZIKA - RAČUNSKE VEŽBE - I NEDELJA

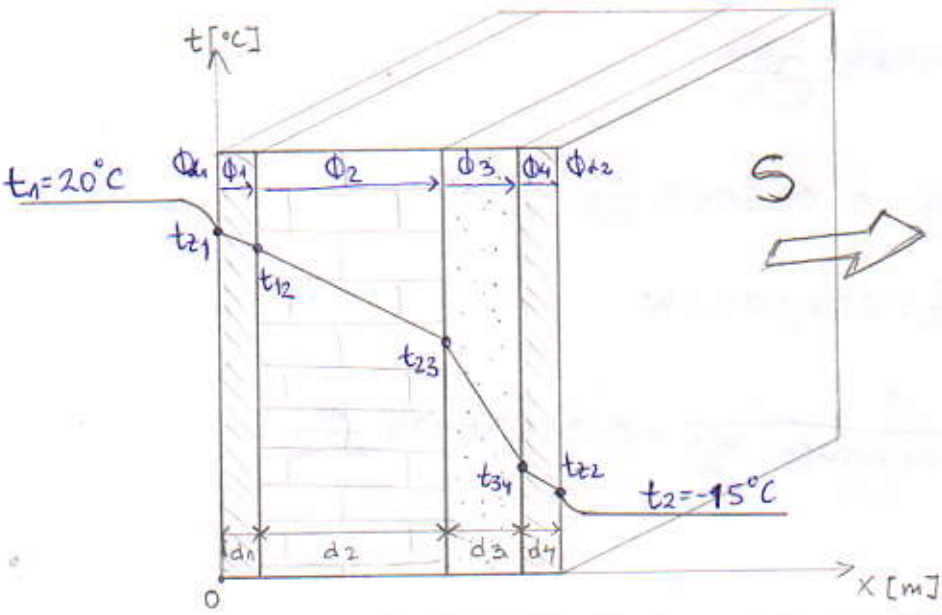
① Ravan fasadni zid, površine $25m^2$, koji je deo termičkog omotača zgrade, sastoji se od 4 sloja. Debljine i toplotne provodnosti pojedinih slojeva su date u donjoj tabeli. Prvi sloj u donjoj tabeli se nalazi sa unutrašnje strane zida, dok se poslednji sloj nalazi sa spoljašnje strane. Koeficijenti prelaza toplote sa unutrašnje i spoljašnje strane zida su 7,69 $W/K{\cdot}m^2$ i 25 $W/K{\cdot}m^2$ respektivno. Spoljašnja temperatura je $-15°C$, a temperatura unutar prostorije je $20°C$. Ako su termalni kontakti između slojeva idealni odrediti:

a) Gustinu toplotnog fluksa, toplotni fluks i koeficijent prolaza toplote
b) Temperaturu zida sa spoljašnje i unutrašnje strane
c) Temperature na spojevima materijala
d) Temperatursko polje unutar kamene vune i giter bloka

Br.	Materijal	Debljina d [cm]	$\lambda\left[\frac{W}{mK}\right]$
1	Malter (unutra)	2	1.40
2	Giter blok	30	0.61
3	Kamena vuna	8	0.034
4	Malter (spolja)	2	1.40

λ - koeficijent toplotne provodnosti

a)



$\alpha_1 = 7{,}69 \frac{W}{m^2K}$

$\alpha_2 = 25 \frac{W}{m^2K}$

Φ - toplotni fluks [W]

q - gustina toplotnog fluksa $\left[\frac{W}{m^2}\right]$

U - koeficijent prolaza toplote $\left[\frac{W}{m^2K}\right]$

$$\Phi = \frac{t_1 - t_2}{R_T} = \frac{t_1 - t_2}{\frac{1}{\alpha_1 \cdot S} + \frac{d_1}{\lambda_1 S} + \frac{d_2}{\lambda_2 S} + \frac{d_3}{\lambda_3 S} + \frac{d_4}{\lambda_4 S} + \frac{1}{\alpha_2 \cdot S}} \Rightarrow \Phi = \frac{\Delta t}{R_T}$$

($d_1 = d_4$, $\lambda_1 = \lambda_4$)

$\frac{1}{\alpha_1 \cdot S}$ — otpor prelaska toplote sa vazduha na zid

$\frac{d_1}{\lambda_1 S}$ — otpor kroz malter

R_T - termički otpor $\left(\frac{1}{\alpha \cdot S}; \frac{d}{\lambda \cdot S}\right)$ $\left[\frac{K}{W}\right]$

$$q = \frac{\Phi}{S} = \frac{t_1 - t_2}{\frac{1}{\alpha_1} + \frac{2 \cdot d_1}{\lambda_1} + \frac{d_2}{\lambda_2} + \frac{d_3}{\lambda_3} + \frac{1}{\alpha_2}} \Rightarrow q = \frac{\Delta t}{R}$$

R – „R" („er") vrednost zida, $R = S \cdot R_T$ $\left[\frac{K \cdot m^2}{W}\right]$

$$R_{\alpha_1} = \frac{1}{\alpha_1} = \frac{1}{7{,}69 \frac{W}{m^2 K}} = 0{,}130039 \frac{m^2 K}{W}$$

$$R_{\alpha_2} = \frac{1}{\alpha_2} = 0{,}04 \frac{m^2 K}{W}$$

$$R_1 = \frac{d_1}{\lambda_1} = \frac{0{,}02\,m}{1{,}40 \frac{W}{mK}} = 0{,}0142857 \frac{m^2 K}{W}$$

$$R_2 = \frac{d_2}{\lambda_2} = \frac{0{,}13\,m}{0{,}61 \frac{W}{mK}} = 0{,}491803278 \frac{m^2 K}{W}$$

$$R_3 = \frac{d_3}{\lambda_3} = \frac{0{,}08\,m}{0{,}034 \frac{W}{mK}} = 2{,}352941176 \frac{m^2 K}{W}$$

$$R_4 = \frac{d_4}{\lambda_4} = \frac{0{,}02\,m}{1{,}40 \frac{W}{mK}} = 0{,}0142857 \frac{m^2 K}{W}$$

$$R = R_{\alpha_1} + \sum_{i=1}^{4} R_i + R_{\alpha_2} = 3{,}043354895 \frac{m^2 K}{W}$$

$$q = \frac{t_1 - t_2}{R} = \frac{20°C - (-15°C)}{3{,}043354895 \frac{m^2 K}{W}} = 11{,}500466617 \frac{W}{m^2}$$

$$\Phi = S \cdot q = 25 m^2 \cdot 11{,}500466617 \frac{W}{m^2} = 287{,}5116\,W$$

$$U = \frac{1}{R} = \frac{1}{R_{\alpha_1} + \sum_{i=1}^{4} R_i + R_{\alpha_2}} = \frac{1}{3{,}043354895 \frac{m^2 K}{W}} = 0{,}328584475 \frac{W}{m^2 K}$$

$q = 11{,}5005 \frac{W}{m^2}$ $\quad \Phi = 287{,}5116\,W$ $\quad U = 0{,}3286 \frac{W}{m^2 K}$

b) $\Phi = \alpha_1 \cdot S\,(t_1 - t_{z1})$

$\Phi = \alpha_2 \cdot S\,(t_{z2} - t_2)$

fluks prelaza toplote između vazduha i zida i zida i vazduha

$$\Phi_\lambda = \frac{\lambda \cdot S}{d}\,(t_2 - t_1)$$

provođenje kroz zid

Furijeov zakon provođenja toplote

$$t_{z1} = t_1 - \frac{\Phi}{\alpha_1 S} = t_1 - \frac{q}{\alpha_1} = 18{,}5045\ ^\circ C$$

$$t_{z2} = t_2 + \frac{\Phi}{\alpha_2 \cdot S} = t_2 + \frac{q}{\alpha_2} = -14{,}53998\ ^\circ C$$

c) $\Phi = \frac{\lambda_1 S}{d_1}\,(t_{z1} - t_{12})$

$$t_{12} = t_{z1} - \frac{\Phi}{S}\cdot\frac{d_1}{\lambda_1} = \underbrace{t_1 - \frac{q}{\alpha_1}}_{t_{z1}} - q\cdot\frac{d_1}{\lambda_1} = t_1 - q\left(\frac{1}{\alpha_1} + \frac{d_1}{\lambda_1}\right) = 18{,}3402\ ^\circ C$$

$\Phi = \frac{\lambda_2 S}{d_2}\,(t_{12} - t_{23})$

$$t_{23} = t_{12} - \frac{\Phi}{S}\frac{d_2}{\lambda_2} = t_1 - q\left(\frac{1}{\alpha_1} + \frac{d_1}{\lambda_1}\right) - q\,\frac{d_2}{\lambda_2} = t_1 - q\left(\frac{1}{\alpha_1} + \frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2}\right) = 12{,}6842\ ^\circ C$$

$\Phi = \frac{\lambda_3 S}{d_3}\,(t_{23} - t_{34})$

$$t_{34} = t_{23} - \frac{\Phi}{S}\frac{d_3}{\lambda_3} = t_1 - q\left(\frac{1}{\alpha_1} + \frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2}\right) - q\cdot\frac{d_3}{\lambda_3} = t_1 - q\left(\frac{1}{\alpha_1} + \frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2} + \frac{d_3}{\lambda_3}\right) = -14{,}3758\ ^\circ C$$

d) $t_2(x),\ t_3(x) = ?$ [na predavanjima su izvođeni izrazi za temperatursko polje unutar slojeva]

$$t_2(x) = t_{12} - \frac{t_{12} - t_{23}}{d_2}\,(x - d_1) = t_1 - \frac{t_1 - t_2}{\frac{1}{\alpha_1} + \sum_{i=1}^{1}\frac{d_i}{\lambda_i} + \frac{1}{\alpha_2}}\left(\frac{1}{\alpha_1} + \frac{d_1}{\lambda_1} + \frac{x - d_1}{\lambda_2}\right)$$

ili

$$t_2(x) = t_{12} - \frac{t_{12} - t_{23}}{d_2}\,(x - d_1) = 18{,}3402\ ^\circ C - 18{,}853\ \frac{^\circ C}{m}\,(x - 0{,}02\,m)$$

$$t_3(x) = t_{23} - \frac{t_{23} - t_{34}}{d_3}\,(x - (d_1 + d_2)) = t_1 - \frac{t_1 - t_2}{\frac{1}{\alpha_1} + \sum_{i=1}^{2}\frac{d_i}{\lambda_i} + \frac{1}{\alpha_2}}\left(\frac{1}{\alpha_1} + \frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2} + \frac{x - (d_1 + d_2)}{\lambda_3}\right)$$

ili

$$t_3(x) = t_{23} - \frac{t_{23} - t_{34}}{d_3}\,(x - d_1 - d_2) = 12{,}6842\ ^\circ C - 338{,}25\ \frac{^\circ C}{m}\,(x - 0{,}32\,m)$$

(2.) Cilindrični fasadni zid, visine 20m, koji je deo termičkog omotača zgrade, sastoji se od 4 sloja. Unutrašnji poluprečnik zida je 4m. Debljin i toplotne provodnosti pojedinih slojeva su date u donjoj tabeli. Prvi sloj u donjoj tabeli se nalazi sa unutrašnje strane zida, dok se poslednji sloj nalazi sa spoljašnje strane. koeficijenti prelaza toplote sa unutrašnje i spoljašnje strane zida su $7,69 \frac{W}{K \cdot m^2}$ i $25 \frac{W}{K \cdot m^2}$ respektivno. Spoljašnja temperatura je $-12°C$, a temperatura unutar prostorije je $20°C$. Ako su termalni kontakti između slojeva idealni, odrediti:

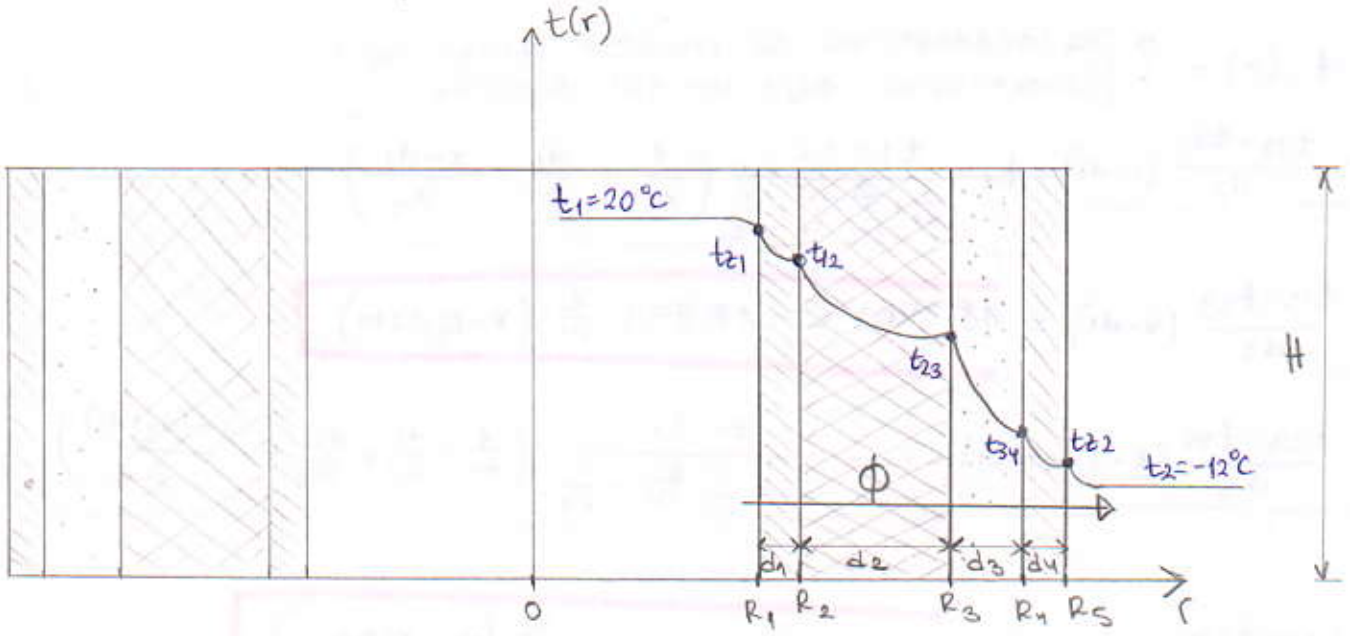
a) Podužnu gustinu toplotnog fluksa i toplotni fluks

b) Temperaturu zida sa spoljašnje i unutrašnje strane

c) Temperature na spojevima materijala

d) Temperatursko polje unutar armiranog betona i ekspandiranog polistirena

Br.	Materijal	Debljina d [m]	$\lambda \left[\frac{W}{mK}\right]$
1	Produžni malter (unutra)	2	0,99
2	Armirani beton	30	2,33
3	Ekspandirani polistiren	8	0,041
4	Produžni malter (spolja)	2	0,99



$\alpha_1 = 7,69 \frac{W}{K \cdot m^2}$ $\alpha_2 = 25 \frac{W}{K \cdot m^2}$

$R_1 = 4m$ *

$R_2 = R_1 + d_1 = 4{,}02\,m$

$R_3 = R_2 + d_2 = 4{,}32\,m$

$R_4 = R_3 + d_3 = 4{,}4\,m$

$R_5 = R_4 + d_4 = 4{,}42\,m$

* obratite pažnju da li je u zadatku dat prečnik ili poluprečnik zida! (R_i - poluprečnik)

a) q_h - podužna gustina toplotnog fluksa $\left[\frac{W}{m}\right]$

$$q_h = \frac{\Phi}{H} = \frac{t_1 - t_2}{\frac{1}{2\pi}\left(\frac{1}{\alpha_1 R_1} + \sum_{i=1}^{4}\frac{1}{\lambda_i}\ln\frac{R_{i+1}}{R_i} + \frac{1}{\alpha_2 R_5}\right)} = 379{,}6427\ \frac{W}{m}$$

$$q_h = \frac{\Phi}{H} \Rightarrow \Phi = q_h \cdot H = 379{,}6427\ \frac{W}{m} \cdot 20\,m = 7592{,}854\,W$$

b)

$$\Phi_{\alpha_1} = \alpha_1 S_1 (t_1 - t_{z1}) = \Phi$$

$$\Phi_{\alpha_2} = \alpha_2 S_2 (t_{z2} - t_2) = \Phi$$

$$t_{z1} = t_1 - \frac{\Phi}{\alpha_1 \cdot 2\pi R_1 H} = t_1 - \frac{q_h}{\alpha_1 2\pi R_1} = 18{,}035695\,°C$$

$$t_{z2} = t_2 + \frac{\Phi}{\alpha_2 2\pi R_5 H} = t_2 + \frac{q_h}{\alpha_2 2\pi R_5} = -11{,}4532\,°C$$

c)

$$\Phi_1 = \frac{t_{z1} - t_{12}}{\frac{1}{2\pi\lambda_1 H}\cdot\ln\left(\frac{R_2}{R_1}\right)}$$

$$\Rightarrow t_{12} = t_{z1} - \Phi \cdot \frac{1}{H}\cdot\frac{1}{2\pi\cdot\lambda_1}\ln\left(\frac{R_2}{R_1}\right) = t_1 - \frac{t_1 - t_2}{\frac{1}{\alpha_1 R_1} + \sum_{i=1}^{4}\frac{1}{\lambda_i}\ln\frac{R_{i+1}}{R_i} + \frac{1}{\alpha_2 R_5}}\left(\frac{1}{\alpha_1 R_1} + \frac{1}{\lambda_1}\ln\left(\frac{R_2}{R_1}\right)\right)$$

$$t_{12} = t_{z1} - \frac{q_h}{2\pi}\cdot\frac{1}{\lambda_1}\ln\left(\frac{R_2}{R_1}\right) = 17{,}73129\,°C$$

$$t_{23} = t_1 - \frac{q_h}{2\pi}\left(\frac{1}{\alpha_1 R_1} + \frac{1}{\lambda_1}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{\lambda_2}\ln\left(\frac{R_3}{R_2}\right)\right) = t_{12} - \frac{q_h}{2\pi}\cdot\frac{1}{\lambda_2}\ln\left(\frac{R_3}{R_2}\right)$$

$$t_{23} = 15{,}8649\,°C$$

$$t_{34} = t_1 - \frac{q_h}{2\pi}\left(\frac{1}{\alpha_1 R_1} + \frac{1}{\lambda_1}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{\lambda_2}\ln\left(\frac{R_3}{R_2}\right) + \frac{1}{\lambda_3}\left(\frac{R_4}{R_3}\right)\right) = t_{23} - \frac{q_h}{2\pi}\cdot\frac{1}{\lambda_3}\ln\left(\frac{R_4}{R_3}\right)$$

$$t_{34} = -11{,}1764\,°C$$

d) *formula iz lekcije „Provođenje toplote kroz homogeni zid"

$$t_2(r) = t_{12} - \frac{\Phi}{2\pi r_2 H} \ln\frac{r}{R_2} = t_1 - \frac{t_1 - t_2}{\frac{1}{\lambda_1 R_1} + \sum_{i=1}^{4} \frac{1}{\lambda_i} \ln\frac{R_{iH}}{R_i} + \frac{1}{\lambda_2 R_5}} \left(\frac{1}{\lambda_1 R_1} + \frac{1}{\lambda_1} \ln\left(\frac{R_2}{R_1}\right) + \frac{1}{\lambda_2} \ln\left(\frac{r}{R_2}\right)\right.$$

↳ formula za prolaz toplote višeslojnog zida

$$t_2(r) = t_{12} - \frac{q_u}{2\pi} \cdot \frac{1}{\lambda_2} \ln\left(\frac{r}{R_2}\right) = 17,73129\,°C - 25,932\,°C \cdot \ln\left(\frac{r}{4,02}\right)$$

$$t_3(r) = t_{23} - \frac{\Phi}{2\pi r_3 H} \ln\left(\frac{r}{R_3}\right) = t_1 - \frac{t_1 - t_2}{\frac{1}{\lambda_1 R_1} + \sum_{i=1}^{4} \frac{1}{\lambda_i} \ln\left(\frac{R_{iH}}{R_i}\right) + \frac{1}{\lambda_2 R_5}} \left(\frac{1}{\lambda_1 R_1} + \frac{1}{\lambda_1} \ln\frac{R_2}{R_1} + \frac{1}{\lambda_2} \ln\frac{R_3}{R_2} + \frac{1}{\lambda_3} \ln\frac{r}{R_3}\right)$$

$$t_3(r) = t_{23} - \frac{q_u}{2\pi} \frac{1}{\lambda_3} \ln\left(\frac{r}{R_3}\right) = 15,8649\,°C - 1473,7076\,°C \ln\left(\frac{r}{4,32}\right)$$

*NAPOMENA: Međurezultate zaokružujte na 5 decimala, a krajnje rezultate na dve.
Pazite na račun; ukoliko bilo gde u zadatku napravite grešku koja utiče na tačnost rezultata, ceo zadatak se smatra netačnim!

saradnik Milica Radojičić