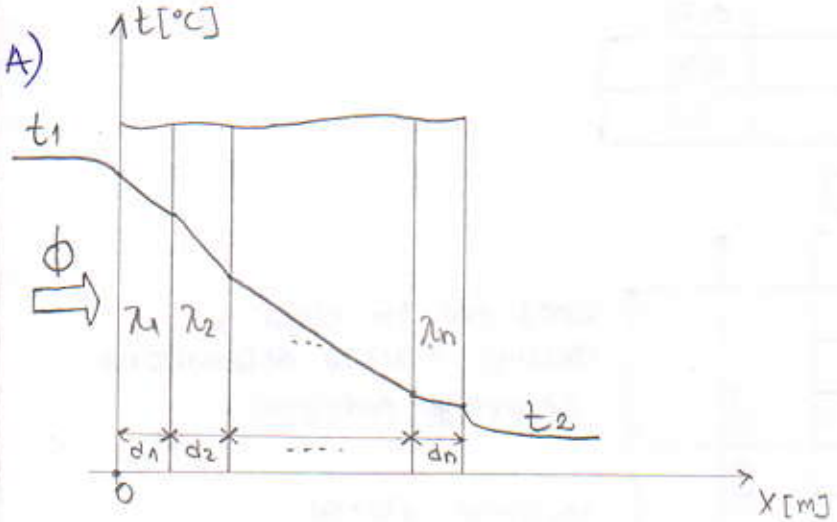
GRAĐEVINSKA FIZIKA – RAČUNSKE VEŽBE – II NEDELJA

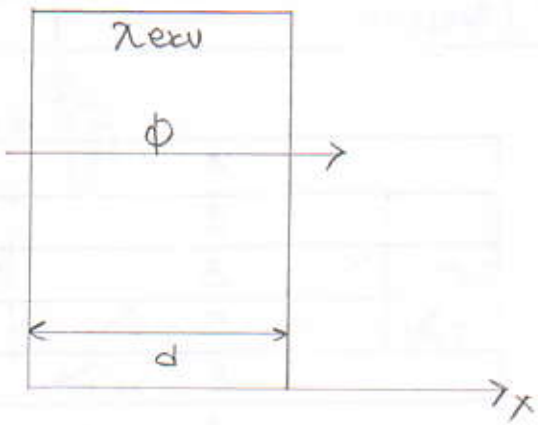
(1.) A) Ravan nehomogen zid dimenzija axb se sastoji od n slojeva debljina d_i, $i = 1, \dots, n$. Zid je potrebno zameniti homogenim zidom iste debljine. Naći ekvivalentnu toplotnu provodnost materijala homogenog zida tako da fluks toplotnih gubitaka nehomogenog i homogenog zida bude isti.

B) Ravan zid se sastoji od 4 sloja. Debljine i toplotne provodnosti slojeva su date u tabeli. Ako je spoljašnja temperatura vazduha 5°C, a unutrašnja 18°C, naći toplotnu provodnost ekvivalentnog jednoslojnog zida tako da fluks toplotnih gubitaka jednoslojnog i višeslojnog zida bude isti.

Br.	Materijal	Debljina d [cm]	$\lambda \left[\frac{W}{mK}\right]$
1	Malter	1,5	1,4
2	Armirani beton	30	2,33
3	Staklena vuna	6	0,032
4	Produžni malter	1	0,99

A)





$$\Phi_1 = \frac{t_1 - t_2}{\sum R_{Ti}} = \frac{t_1 - t_2}{\frac{1}{\alpha_1 S} + \sum_{i=1}^{n} \frac{d_i}{\lambda_i \cdot S} + \frac{1}{\alpha_2 S}} \; [W] \qquad \Phi_2 = \frac{t_1 - t_2}{R_{Tekv}} = \frac{t_1 - t_2}{\frac{1}{\alpha_1 S} + \frac{d}{\lambda_{ekv} S} + \frac{1}{\alpha_2 S}}$$

$$d = d_1 + d_2 + \dots + d_n \qquad \Phi_1 = \Phi_2$$

$$\frac{t_1 - t_2}{\frac{1}{\alpha_1 S} + \frac{1}{S}\sum_{i=1}^{n} \frac{d_i}{\lambda_i} + \frac{1}{\alpha_2 S}} = \frac{t_1 - t_2}{\frac{1}{\alpha_1 S} + \frac{d}{\lambda_{ekv} S} + \frac{1}{\alpha_2 S}} \Rightarrow \sum_{i=1}^{n} \frac{d_i}{\lambda_i} = \frac{d}{\lambda_{ekv}}$$

$$\boxed{\lambda_{ekv} = \frac{d}{\sum_{i=1}^{n} \frac{d_i}{\lambda_i}} = \frac{\sum d_i}{\sum \frac{d_i}{\lambda_i}} = \frac{d}{\left(\frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2} + \dots + \frac{d_n}{\lambda_n}\right)}}$$

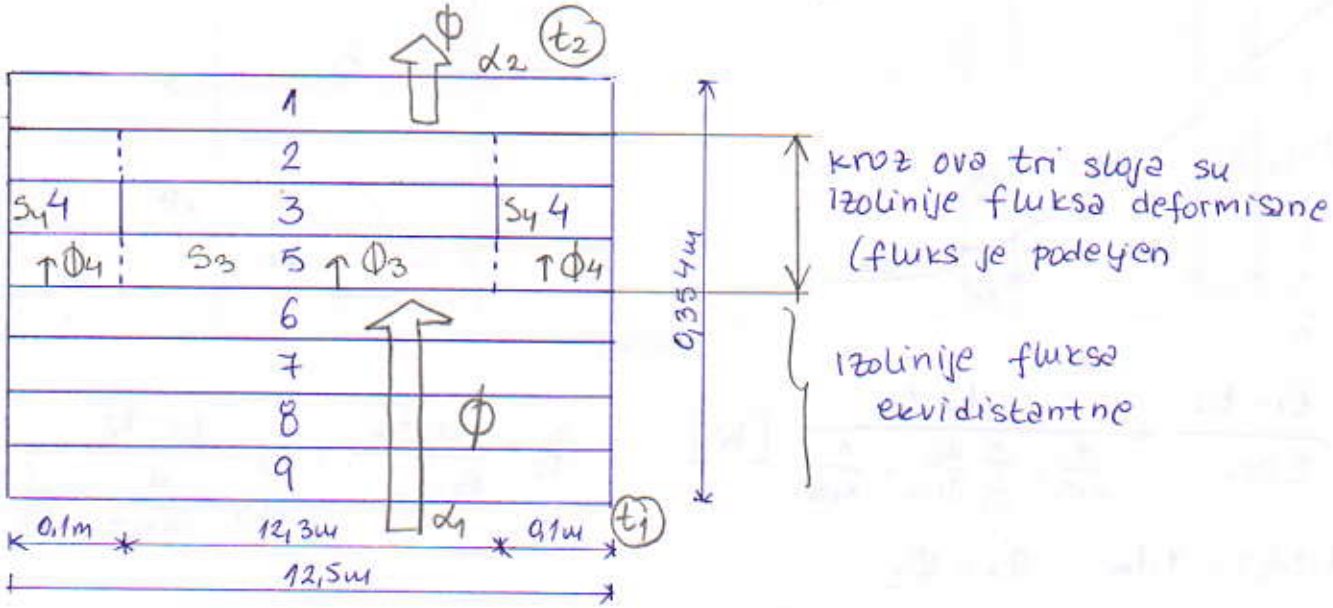
b) $$\lambda_{ekv} = \frac{\sum_{i=1}^{4} d_i}{\sum_{i=1}^{4} \frac{d_i}{\lambda_i}} = \frac{d_1+d_2+d_3+d_4}{\frac{d_1}{\lambda_1}+\frac{d_2}{\lambda_2}+\frac{d_3}{\lambda_3}+\frac{d_4}{\lambda_4}} = 0,190164 \frac{W}{mK}$$

2. Izračunati termički otpor prolaza toplote, koeficijent prolaza toplote i R-vrednost za ravan višeslojni zid, dužine 30m i širine 12,5m čije su dimenzije i poprečni presek prikazani na slici. Zid je krov i deo je termičkog omotača zgrade. Koeficijenti prelaza toplote sa unutrašnje i spoljašnje strane zida su 10 $^{W}/_{m^2k}$ i 25 $^{W}/_{m^2k}$ respektivno. Toplotne provodnosti za pojedine materijale su date u tabeli. Prvi sloj u tabeli se nalazi sa spoljašnje strane zida, a poslednji sa unutrašnje.

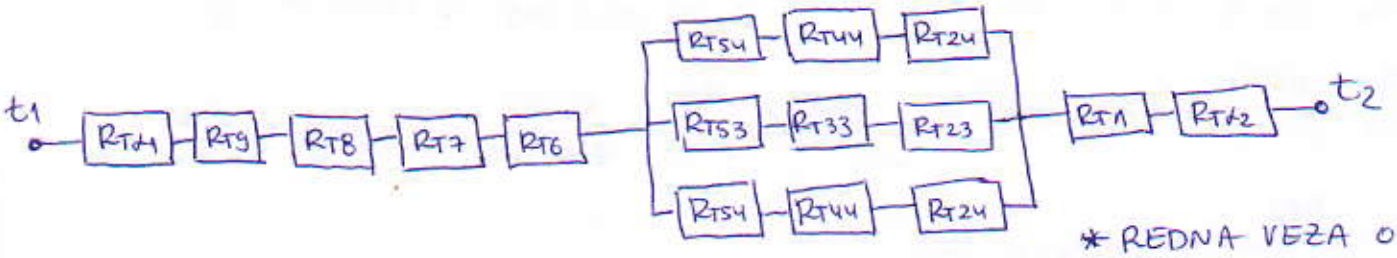
Br.	Materijal	Debljina d[cm]	$\lambda[\frac{W}{mK}]$
1	Lepljena PVC membrana	0,5	0,19
2	Geotekstil	0,5	0,19
3	Duropor	12	0,035
4	Drvo - bor	12	0,14
5	Parna brana	0,4	0,19
6	Sloj za izravnavanje pritiska	2	0,81
7	Sloj za pad	2	0,76
8	AB ploča	16	2,33
9	Malter	2	1,4

$\alpha_1 = 10 \frac{W}{m^2k}$

$\alpha_2 = 25 \frac{W}{m^2k}$



→ ŠEMA OTPORNIKA (OBAVEZNO NACRTATI !)

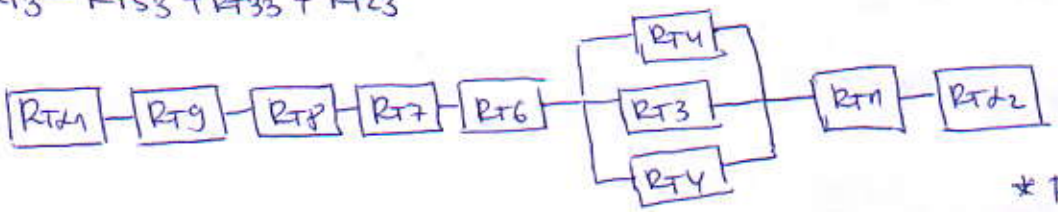


* REDNA VEZA OTPORNIKA:

$$R_{ekv} = \sum_i R_i$$

$R_{T4} = R_{T54} + R_{T44} + R_{T24}$

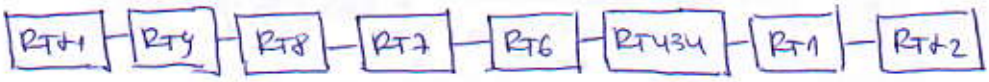
$R_{T3} = R_{T53} + R_{T33} + R_{T23}$



* PARALELNA VEZA OTPORNIKA:

$$\frac{1}{R_{ekv}} = \sum_i \frac{1}{R_i}$$

$$\frac{1}{R_{T434}} = \frac{1}{R_{T4}} + \frac{1}{R_{T3}} + \frac{1}{R_{T4}} \Rightarrow R_{T434} = \frac{R_{T4} \cdot R_{T3}}{2R_{T3} + R_{T4}}$$



$R_T = R_{Td_1} + R_{Tg} + R_{T8} + R_{T7} + R_{T6} + R_{T343} + R_{T1} + R_{Td_2}$

→ ukupna površina zida:

$S = 30 \cdot 12{,}5 = 375\ m^2$

→ površina zida sa duroporom:

$S_3 = 30 \cdot 12{,}3 = 369\ m^2$

→ površina zida sa drvetom:

$S_4 = 30 \cdot 0{,}1 = 3\ m^2$

$$R_{Td_i} = \frac{1}{\alpha_i S}$$

$$R_{Ti} = \frac{d_i}{\lambda_i \cdot S}$$

$$R_{Tij} = \frac{d_i}{\lambda_i \cdot S_j}$$

$$R_{Td_1} = \frac{1}{\alpha_1 \cdot S} = \frac{1}{10 \frac{W}{m^2 K} \cdot 375 m^2} = 2{,}6 \cdot 10^{-4} \frac{K}{W}$$

$$R_{Td_2} = \frac{1}{\alpha_2 S} = \frac{1}{25 \frac{W}{m^2 K} \cdot 375 m^2} = 1{,}06 \cdot 10^{-4} \frac{K}{W}$$

$$R_{T1} = \frac{d_1}{\lambda_1 \cdot S} = \frac{0{,}005\ m}{0{,}19 \frac{W}{mK} \cdot 375 m^2} = 7{,}01754 \cdot 10^{-5} \frac{K}{W}$$

$$R_{T24} = \frac{d_2}{\lambda_2 S_4} = \frac{0{,}005}{0{,}19 \cdot 3} = 8{,}77193 \cdot 10^{-3} \frac{K}{W}$$

$$R_{T23} = \frac{d_2}{\lambda_2 S_3} = \frac{0{,}005}{0{,}19 \cdot 369} = 7{,}13165 \cdot 10^{-5} \frac{K}{W}$$

$$R_{T44} = \frac{d_4}{\lambda_4 S_4} = \frac{0,12}{0,14 \cdot 3} = 0,28571 \frac{K}{W} \qquad R_{T33} = \frac{d_3}{\lambda_3 S_3} = \frac{0,12}{0,035 \cdot 369} = 9,29152 \cdot 10^{-3} \frac{K}{W}$$

$$R_{T54} = \frac{d_5}{\lambda_5 S_4} = \frac{0,004}{0,19 \cdot 3} = 7,01754 \cdot 10^{-3} \frac{K}{W} \qquad R_{T53} = \frac{d_5}{\lambda_5 S_3} = \frac{0,004}{0,19 \cdot 369} = 5,70532 \cdot 10^{-5} \frac{K}{W}$$

$$R_{T6} = \frac{d_6}{\lambda_6 S} = \frac{0,02}{0,81 \cdot 375} = 6,58436 \cdot 10^{-5} \frac{K}{W}$$

$$R_{T7} = \frac{d_7}{\lambda_7 \cdot S} = \frac{0,02}{0,76 \cdot 375} = 7,01754 \cdot 10^{-5} \frac{K}{W}$$

$$R_{T8} = \frac{d_8}{\lambda_8 S} = \frac{0,16}{2,33 \cdot 375} = 1,83119 \cdot 10^{-4} \frac{K}{W}$$

$$R_{T9} = \frac{d_9}{\lambda_9 \cdot S} = \frac{0,02}{1,4 \cdot 375} = 3,80952 \cdot 10^{-5} \frac{K}{W}$$

$$\left.\begin{aligned} R_{T4} &= R_{T54} + R_{T24} + R_{T44} = 0,301504 \frac{K}{W} \\ R_{T3} &= R_{T53} + R_{T23} + R_{T33} = 9,41989 \cdot 10^{-3} \frac{K}{W} \end{aligned}\right\} R_{T434} = \frac{R_{T4} \cdot R_{T3}}{2R_{T3} + R_{T4}} = 8,865896 \cdot 10^{-3} \frac{K}{W}$$

$$R_T = R_{Td1} + R_{T9} + R_{T8} + R_{T7} + R_{T6} + R_{T434} + R_{T1} + R_{Td2} = \boxed{9,6666 \cdot 10^{-3} \frac{K}{W}}$$

R-vrednost $\quad R = R_T \cdot S$

$$R = (R_{Td1} + R_{T9} + R_{T8} + R_{T7} + R_{T6} + R_{T1} + R_{Td2}) \cdot S + R_{434}$$

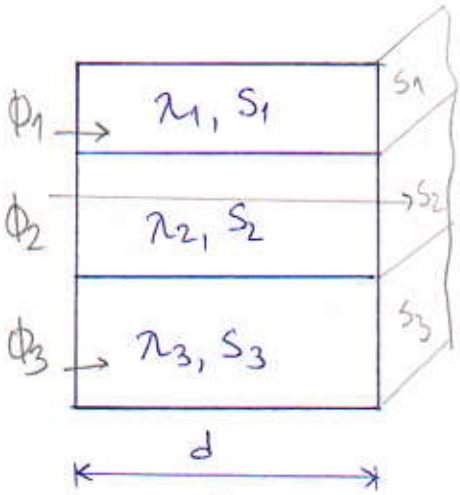
$$R_{434} = \frac{R_4 \cdot R_3}{2R_3 + R_4} \qquad R_4 = R_{T4} \cdot S_4 = 0,904511 \frac{km^2}{W} \qquad R_3 = R_{T3} \cdot S_3 = 3,475939 \frac{km^2}{W}$$

$$R_{434} = 0,400187 \frac{km^2}{W}$$

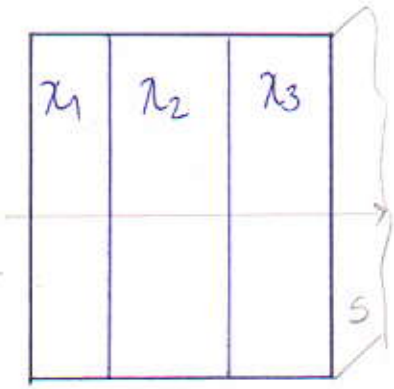
$$R = 8,00744 \cdot 10^{-4} \cdot 375 + 0,400187 = \boxed{0,70047 \frac{k \cdot m^2}{W}}$$

$$U = \frac{1}{R} = \frac{1}{0,70047} = \boxed{1,42762 \frac{W}{K \cdot m^2}}$$

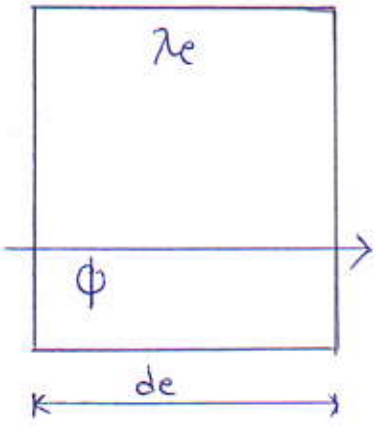
(3.) Ravan heterogen zid dužine L i visine H se sastoji od 3 sloja toplotnih provodnosti $\lambda_1, \lambda_2, \lambda_3$ i prikazan je na slikama a) i b) u poprečnom preseku. Naći ekvivalentne debljine de homogenih zidova za svaki od njih, ako je poznata toplotna provodnost λ_e, pod uslovom da su fluksevi toplotnih gubitaka zidova a) i b) i njima odgovarajućih homogenih zidova isti. Zanemariti prelaze toplote sa vazduha na zid i zida na razduh.



a)



b)



a)

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3$$

$$\Phi = \frac{\Delta t}{\frac{d}{\lambda_1 S_1}} + \frac{\Delta t}{\frac{d}{\lambda_2 S_2}} + \frac{\Delta t}{\frac{d}{\lambda_3 S_3}} = \frac{\Delta t}{\frac{de}{\lambda_e \cdot S}}$$

$$\frac{1}{\frac{de}{\lambda_e S}} = \frac{1}{\frac{d}{\lambda_1 S_1}} + \frac{1}{\frac{d}{\lambda_2 S_2}} + \frac{1}{\frac{d}{\lambda_3 S_3}}$$

$$\frac{\lambda_e \cdot S}{de} = \frac{\lambda_1 S_1 + \lambda_2 S_2 + \lambda_3 S_3}{d} \Rightarrow de = \frac{\lambda_e \cdot d \cdot S}{\lambda_1 S_1 + \lambda_2 S_2 + \lambda_3 S_3}$$

$$de = \lambda_e \cdot \frac{(S_1 + S_2 + S_3) \cdot d}{\lambda_1 S_1 + \lambda_2 S_2 + \lambda_3 S_3}$$

$$* \quad \frac{\lambda_1 S_1 + \lambda_2 S_2 + \lambda_3 S_3}{S_1 + S_2 + S_3} = \lambda_{sr}$$

b) $$\Phi = \frac{\Delta t}{\left(\frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2} + \frac{d_3}{\lambda_3}\right)\frac{1}{S}} = \frac{\Delta t}{\frac{de}{\lambda_e \cdot S}}$$

$$\frac{\lambda_e}{de} = \frac{1}{\frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2} + \frac{d_3}{\lambda_3}} \Rightarrow de = \lambda_e \left(\frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2} + \frac{d_3}{\lambda_3}\right)$$

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