GRAĐEVINSKA FIZIKA - RAČUNSKE VEŽBE - VII NEDELJA

OSVETLJENJE

E - osvetljenost [lx]

$$\boxed{E \stackrel{def}{=} \frac{\Delta\Phi}{\Delta S}}$$

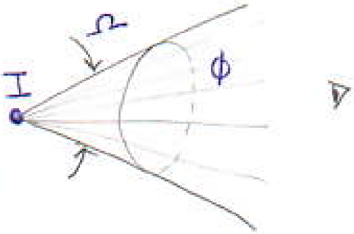
Φ - svetlosni fluks $\left[W = \frac{J}{s}\right]$

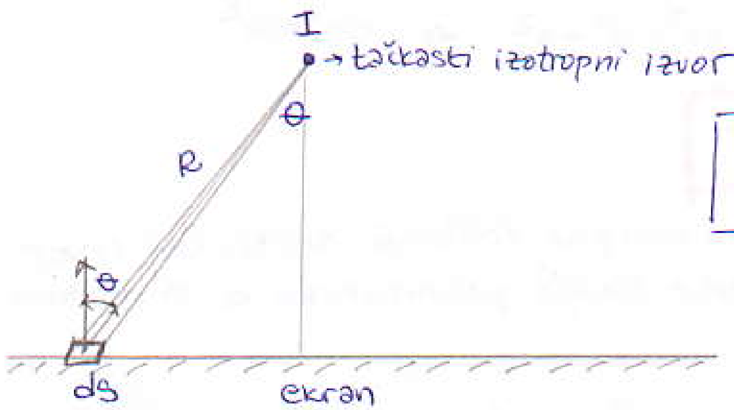
$$E = \frac{d\Phi}{dS} = \frac{I d\Omega}{dS}$$

I - intenzitet svetlosti [cd]

$$\boxed{I \stackrel{def}{=} \frac{d\Phi}{d\Omega}}$$

Ω - prostorni ugao

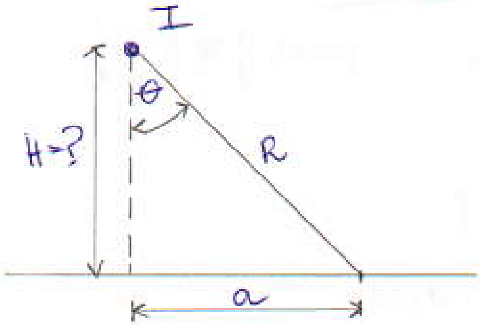




$$\boxed{E = I \cdot \frac{\cos\theta}{R^2}}$$

osvetljenost tačkastog svetlosnog izvora

① Odrediti visinu H tačkastog svetlosnog izvora iznad horizontalne ravni tako da osvetljenost u tački udaljenoj za a od projekcije svetlosnog izvora na horizontalnu ravan bude maksimalna.



$$\cos\theta = \frac{H}{R}$$

$$E = I\cdot\frac{\cos\theta}{R^2} = I\cdot\frac{H}{R^3} = I\cdot\frac{H}{\sqrt{(a^2+H^2)^3}}$$

$$\frac{\partial E}{\partial H} = 0 \quad (\text{maksimalna osvetljenost} \rightarrow \text{izvod} = 0)$$

$$\frac{\partial}{\partial H}\left(\cancel{I}\cdot H\,(a^2+H^2)^{-3/2}\right) = 0$$

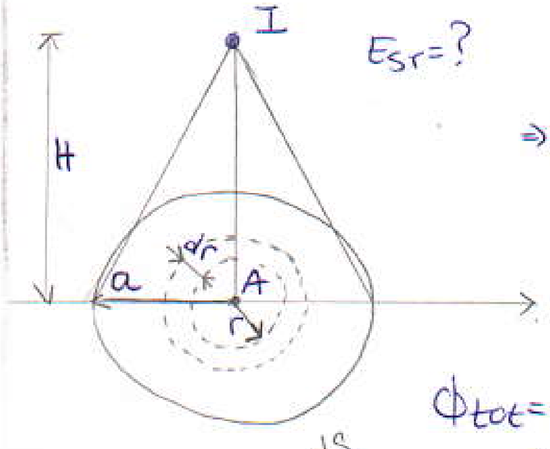
$$1\cdot(a^2+H^2)^{-3/2} + H\left(-\frac{3}{\cancel{2}}\right)(a^2+H^2)^{-\frac{5}{2}}\cdot \cancel{2}H = 0$$

$$\cancel{(a^2+H^2)^{-3/2}}\cdot\left[1 - 3H^2(a^2+H^2)^{-1}\right] = 0$$

$$1 = 3H^2(a^2+H^2)^{-1} \Rightarrow 3H^2 = a^2+H^2 \Rightarrow 2H^2 = a^2$$

$$\boxed{H = \frac{a}{\sqrt{2}}}$$

② Na visini H iznad tačke A nalazi se izotropni tačkasti svetlosni izvor intenziteta I. Odrediti srednju osvetljenost kruga poluprečnika a sa centrom u tački A.

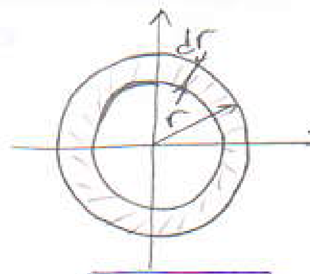


$$E_{sr} = ?$$

$$E \overset{def}{=} \frac{d\phi}{dS} \Rightarrow d\phi = E\,dS$$

$$\Rightarrow \phi_{tot} = \int_S E\,dS$$

$$E_{sr} \overset{def}{=} \frac{\phi_{tot}}{S} = \frac{\phi_{tot}}{\pi a^2}$$



$$\boxed{dS = 2\pi r\,dr}$$

$$\phi_{tot} = \int_S \frac{I\cos\theta}{R^2}\,dS$$

$$E_{sr} = \frac{1}{\pi a^2}\int_S E\cdot\overbrace{2\pi r\,dr}^{dS} = \frac{1}{\pi a^2}\int_{r=0}^{a}\frac{I\cdot H}{R^3}\cdot 2\pi r\,dr$$

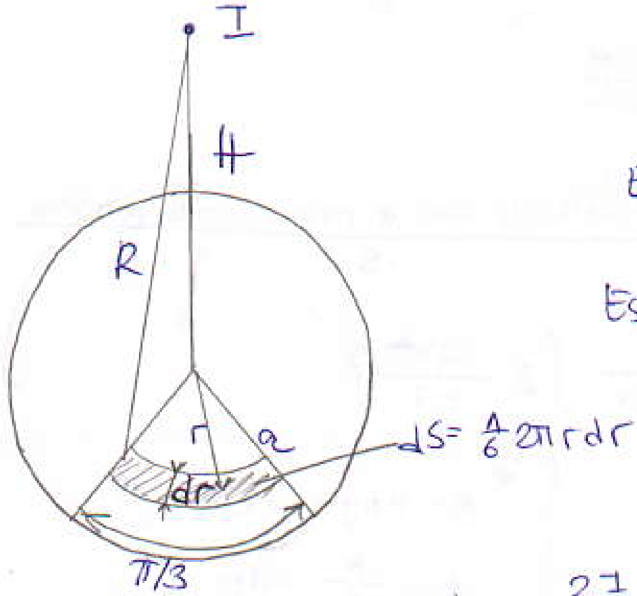
$$E_{sr} = \frac{\cancel{\pi}\cdot I\cdot H}{a^2\cdot\cancel{\pi}}\int_0^a \frac{2r\,dr^{*}}{\sqrt{(r^2+H^2)^3}} = \frac{I\cdot H}{a^2}\int_{H^2}^{a^2+H^2}\frac{dt}{t^{3/2}}$$

$$^{*}\; r^2+H^2 = t$$
$$2r\,dr = dt$$

r	t
0	H^2
a	a^2+H^2

$$E_{sr} = \frac{IH}{a^2}\,\frac{t^{-1/2}}{-1/2}\Bigg|_{H^2}^{a^2+H^2} = \frac{2IH}{a^2}\left[\frac{1}{\sqrt{H^2}} - \frac{1}{\sqrt{a^2+H^2}}\right] = \boxed{\frac{2I}{a^2}\left[1 - \frac{H}{\sqrt{a^2+H^2}}\right]}$$

3. Izračunati srednju osvetljenost isečka poluprečnika 0,2m ako se on osvetljava izvorom jačine 100cd koji se nalazi na visini 1m od centra kruga kome isečak pripada. Ugao $\alpha = \pi/3$



$$E_{sr} = \frac{1}{\frac{\pi a^2}{6}} \int_S E dS = \frac{\cancel{6}}{\pi a^2} \int_S I \cdot \frac{H}{R^3} \cdot \frac{2\pi r dr}{\cancel{6}}$$

$$E_{sr} = \frac{\cancel{\pi} \cdot H \cdot I}{\cancel{\pi} a^2} \int_0^a \frac{2r dr}{\sqrt{(r^2+H^2)^3}}$$

$$\boxed{\begin{matrix} r^2 + H^2 = t \\ 2r dr = dt \end{matrix}}$$

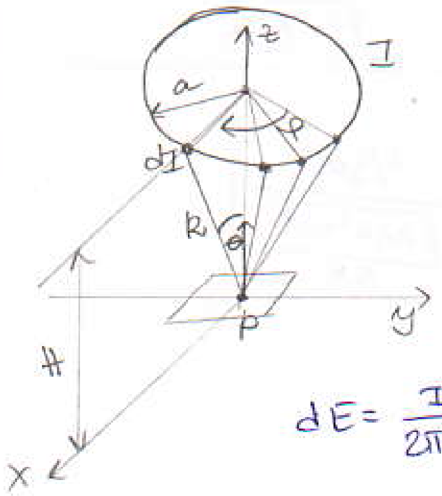
$$E_{sr} = \frac{IH}{a^2} \int_{H^2}^{a^2+H^2} \frac{dt}{t^{3/2}}$$

$$E_{sr} = \frac{I \cdot H}{a^2} \frac{t^{-1/2}}{-1/2} \Big|_{H^2}^{a^2+H^2}$$

$$E_{sr} = \frac{2I}{a^2} \left[1 - \frac{H}{\sqrt{a^2+H^2}} \right]$$

$$E_{sr} = \frac{2 \cdot 100cd}{(0{,}2m)^2} \left[1 - \frac{1m}{\sqrt{(0{,}2m)^2+(1m)^2}} \right] = \boxed{97{,}1\ lx}$$

4. Linijski svetlosni izvor podužnog intenziteta I' je u obliku kružnice poluprečnika a. Kolika je osvetljenost površine paralelne ravni kruga na rastojanju H, u okolini tačke P - projekcije centra kružnice na tu ravan?



$$\boxed{E = I \cdot \frac{\cos\theta}{R^2}} \leftarrow \text{tačkasti izvor}$$

$$dE = dI \frac{\cos\theta}{R^2} \qquad dI = \frac{I \cdot a d\varphi}{2\pi a}$$

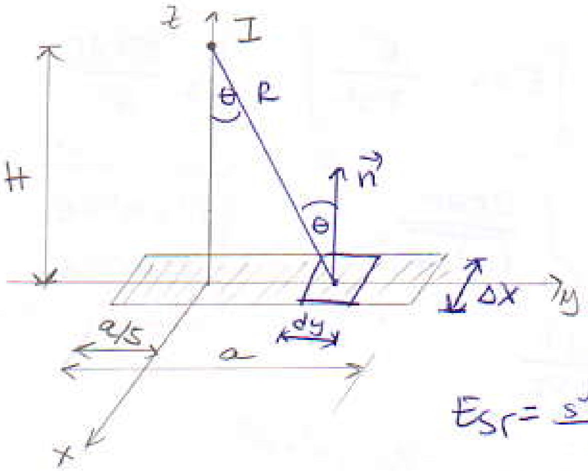
$$\boxed{dI = \underbrace{\left(\frac{I}{2\pi a}\right)}_{I' \text{- podužni intenzitet}} \cdot a d\varphi}$$

$$dE = \frac{I}{2\pi \cancel{a}} \cdot \cancel{a} d\varphi \cdot \frac{\cos\theta}{R^2} = \frac{I}{2\pi} \cdot d\varphi \cdot \frac{H}{\sqrt{(a^2+H^2)^3}}$$

$$E = \int_\varphi dE = \frac{I}{2\pi} \cdot \frac{H}{\sqrt{(a^2+H^2)^3}} \int_0^{2\pi} d\varphi = \frac{I}{\cancel{2\pi}} \cdot \frac{H}{\sqrt{(a^2+H^2)^3}} \cdot \cancel{\varphi} \Big|_0^{2\pi}$$

$$E = I \cdot \frac{H}{\sqrt{(a^2+H^2)^3}} \qquad \boxed{E = \frac{2\pi a I' \cdot H}{\sqrt{(a^2+H^2)^3}}}$$

⑤ Odrediti srednju osvetljenost uske trake dužine a iznad koje se, kao što je prikazano na slici, na visini H nalazi tačkasti izotropni izvor svetlosti, intenziteta I.



$$E = \frac{d\Phi}{dS} \qquad \Phi = \int_S d\Phi = \int_S E\,dS$$

$$E = I \cdot \frac{\cos\theta}{R^2}$$

$$E_{sr} \overset{def}{=} \frac{\overset{(\Phi)}{\text{fluks svetlosti koja je pala na celu površinu}}}{S}$$

$$E_{sr} = \frac{\int_S E\,dS}{S} = \frac{1}{a\cdot\Delta x}\int I\,\frac{\cos\theta\,dS}{R^2}$$

$$E_{sr} = \frac{I}{a\Delta x}\int \frac{H}{R^3}\,\Delta x\cdot dy = \frac{I\cdot H}{a}\int_{-a/5}^{4a/5} \frac{dy}{\sqrt{(H^2+y^2)^3}} \;*$$

$$E_{sr} = \frac{I\cdot \cancel{H}}{a}\int_{t_1}^{t_2} \frac{\cancel{H}/\cancel{\cos^2 t}}{H^{\cancel{3}}/\cos^{\cancel{3}} t}\,dt = \frac{I}{a\cdot H}\int_{t_1}^{t_2} \cos t\,dt$$

$$E_{sr} = \frac{I}{a\cdot H}\sin t\,\Big|_{t_1}^{t_2} = \frac{I}{a\cdot H}\left[\frac{tg\,t}{\sqrt{1+tg^2 t}}\right]\Bigg|_{t_1}^{t_2}$$

$$E_{sr} = \frac{I}{a\cdot H}\cdot\frac{y/H}{\sqrt{1+(y/H)^2}}\Bigg|_{-a/5}^{4a/5}$$

$$E_{sr} = \frac{I}{a\cdot H}\left[\frac{4a/5H}{\sqrt{1+(4a/5H)^2}} + \frac{a/5H}{\sqrt{1+(a/5H)^2}}\right] = \frac{I}{\cancel{a}\cdot H}\cdot\cancel{a}\left[\frac{4/\cancel{5H}}{\frac{\sqrt{25H^2+16a^2}}{\cancel{5H}}} + \frac{1/\cancel{5H}}{\frac{\sqrt{25H^2+a^2}}{\cancel{5H}}}\right]$$

$$\boxed{E_{sr} = \frac{I}{H}\left[\frac{4}{\sqrt{25H^2+16a^2}} + \frac{1}{\sqrt{25H^2+a^2}}\right]}$$

$*$ $y = H\,tg\,t \rightarrow tg\,t = \frac{y}{H}$

$dy = \frac{H}{\cos^2 t}\cdot dt$

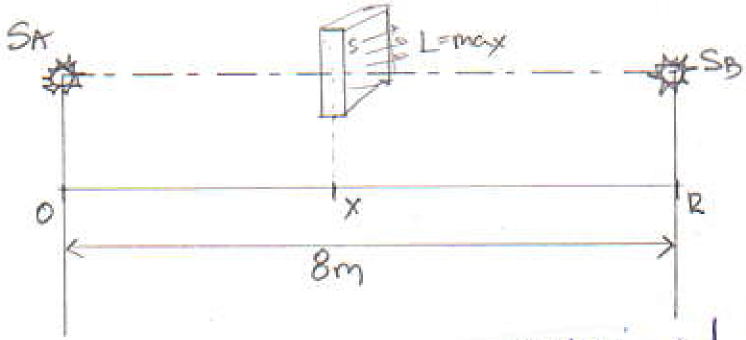
$\sqrt{(y^2+H^2)^3} = \sqrt{(H^2 tg^2 t + H^2)^3} =$

$= H^3\,(1+tg^2 t)^{3/2} =$

$= H^3\left(\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right)^{3/2} = \frac{H^3}{\cos^3 t}$

$\sin t = \frac{tg\,t}{\sqrt{1+tg^2 t}}$

⑥ Dva svetlosna izvora S_A i S_B jačina 400cd i 100cd redom nalaze se na međusobnom rastojanju od 8m. Između njih treba postaviti polutransparentan zaklon, kao na slici, čiji je koeficijent refleksije 0,4, a transmisije 0,3. Na kom nenultom rastojanju od izvora S_B treba postaviti zaklon da bi osvetljaj u okolini tačke D na zaklonu imao maksimalnu vrednost ?

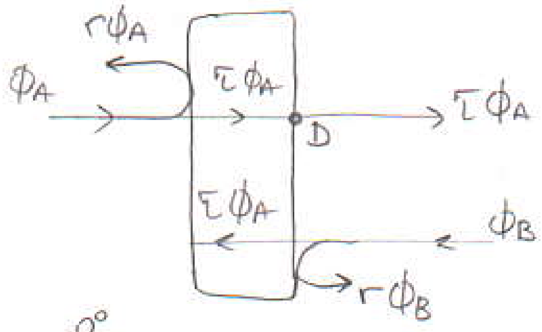


$I_A = 400cd$ $\quad I_B = 100cd$ $\quad r = 0,4$

$\tau = 0,3$

osvetljaj ≠ osvetljenost!

osvetljaj $\leftarrow L \overset{def}{=} \dfrac{\Phi \text{ koji polazi sa površine}}{S}$



$\Phi_{\text{svetlosti koja polazi sa površine}} = \tau\phi_A + r\phi_B$

$$L_D = \frac{\tau\phi_A + r\phi_B}{S} = \tau\cdot\frac{\phi_A}{S} + r\cdot\frac{\phi_B}{S}$$

$$L_D = \tau\cdot E_D^{(A)} + r\cdot E_D^{(B)} = \tau\cdot\frac{I_A\cdot\cos\theta_A}{x^2} + r\,\frac{I_B\cdot\cos\theta_B}{(R-x^2)} \qquad \theta_A \to 0^\circ,\ \theta_B \to 0^\circ$$

$$L_D = \tau\frac{I_A}{x^2} + r\frac{I_B}{(R-x)^2} = \frac{0,3\cdot 400}{x^2} + \frac{0,4\cdot 100}{(R-x)^2}$$

$$\frac{\partial L_D}{\partial x} = 0 \qquad \frac{-2\cdot 120}{x^3} + \frac{2\cdot 40}{(8-x)^3} = 0$$

$$\left(\frac{1}{(8-x)^2}\right)' = \left((8-x)^{-2}\right)' = -2(8-x)^{-3}\cdot(-1) = 2(8-x)^{-3}$$

$$\frac{24}{x^3} = \frac{8}{(8-x)^3}$$

$$8x^3 = 24(8-x)^3 \; / \sqrt[3]{\ }$$

$$2x = \sqrt[3]{24}\,(8-x)$$

$$(2+\sqrt[3]{24})\,x = \sqrt[3]{24}\cdot 8$$

$$x = \frac{\sqrt[3]{24}\cdot 8}{2+\sqrt[3]{24}} = 4,725\text{m}$$

$$\boxed{S_x = (8-4,725) = 3,275\text{m}}$$

saradnik Milica Radojičić