Chapter 25
Composites: fibrous, particulate and foamed

Introduction

The word "composites" has a modern ring. But using the high strength of fibres to stiffen and strengthen a cheap matrix material is probably older than the wheel. The Processional Way in ancient Babylon, one of the lesser wonders of the ancient world, was made of bitumen reinforced with plaited straw. Straw and horse hair have been used to reinforce mud bricks (improving their fracture toughness) for at least 5000 years. Paper is a composite; so is concrete: both were known to the Romans. And almost all natural materials which must bear load – wood, bone, muscle – are composites.

The composite industry, however, is new. It has grown rapidly in the past 30 years with the development of *fibrous composites*: to begin with, *glass-fibre reinforced polymers* (GFRP or fibreglass) and, more recently, *carbon-fibre reinforced polymers* (CFRP). Their use in boats, and their increasing replacement of metals in aircraft and ground transport systems, is a revolution in material usage which is still accelerating.

Composites need not be made of fibres. Plywood is a *lamellar composite*, giving a material with uniform properties in the plane of the sheet (unlike the wood from which it is made). Sheets of GFRP or of CFRP are laminated together, for the same reason. And sandwich panels – composites made of stiff skins with a low-density core – achieve special properties by combining, in a sheet, the best features of two very different components.

Cheapest of all are the *particulate composites*. Aggregate plus cement gives concrete, and the composite is cheaper (per unit volume) than the cement itself. Polymers can be filled with sand, silica flour, or glass particles, increasing the stiffness and wear-resistance, and often reducing the price. And one particulate composite, tungsten-carbide particles in cobalt (known as "cemented carbide" or "hard metal"), is the basis of the heavy-duty cutting tool industry.

But high stiffness is not always what you want. Cushions, packaging and crash-padding require materials with moduli that are lower than those of any solid. This can be done with *foams* – composites of a solid and a gas – which have properties which can be tailored, with great precision, to match the engineering need.

We now examine the properties of fibrous and particulate composites and foams in a little more detail. With these materials, more than any other, properties can be designed-in; the characteristics of the material itself can be engineered.

Fibrous composites

Polymers have a low stiffness, and (in the right range of temperature) are ductile. Ceramics and glasses are stiff and strong, but are catastrophically brittle. In *fibrous*

composites we exploit the great strength of the ceramic while avoiding the catastrophe: the brittle failure of fibres leads to a progressive, not a sudden, failure.

If the fibres of a composite are aligned along the loading direction, then the stiffness and the strength are, roughly speaking, an average of those of the matrix and fibres, weighted by their volume fractions. But not all composite properties are just a linear combination of those of the components. Their great attraction lies in the fact that, frequently, something extra is gained.

The toughness is an example. If a crack simply ran through a GFRP composite, one might (at first sight) expect the toughness to be a simple weighted average of that of glass and epoxy; and both are low. But that is not what happens. The strong fibres pull out of the epoxy. In pulling out, work is done and this work contributes to the toughness of the composite. The toughness is greater – often much greater – than the linear combination.

Polymer-matrix composites for aerospace and transport are made by laying up glass, carbon or Kevlar fibres (Table 25.1) in an uncured mixture of resin and hardener. The resin cures, taking up the shape of the mould and bonding to the fibres. Many composites are based on epoxies, though there is now a trend to using the cheaper polyesters.

Laying-up is a slow, labour-intensive job. It can be by-passed by using thermoplastics containing chopped fibres which can be injection moulded. The random chopped fibres are not quite as effective as laid-up continuous fibres, which can be oriented to maximise their contribution to the strength. But the flow pattern in injection moulding helps to line the fibres up, so that clever mould design can give a stiff, strong product. The technique is used increasingly for sports goods (tennis racquets, for instance) and light-weight hiking gear (like back-pack frames).
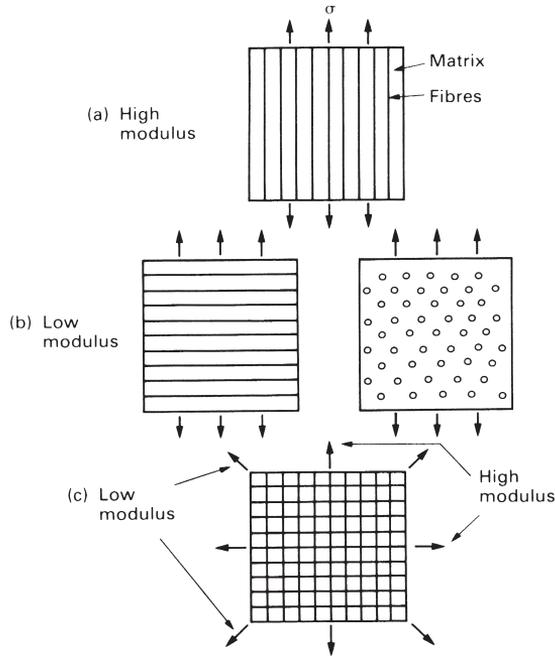
Making good fibre-composites is not easy; large companies have been bankrupted by their failure to do so. The technology is better understood than it used to be; the tricks can be found in the books listed under Further reading. But suppose you can make them, you still have to know how to use them. That needs an understanding of their properties, which we examine next. The important properties of three common composites are listed in Table 25.2, where they are compared with a high-strength steel and a high-strength aluminium alloy of the sort used for aircraft structures.

Table 25.1 Properties of some fibres and matrices

Material	Density ρ (Mg m^{-3})	Modulus E(GPa)	Strength σ_f (MPa)
Fibres			
Carbon, Type1	1.95	390	2200
Carbon, Type2	1.75	250	2700
Cellulose fibres	1.61	60	1200
Glass (E-glass)	2.56	76	1400–2500
Kevlar	1.45	125	2760
Matrices			
Epoxies	1.2–1.4	2.1–5.5	40–85
Polyesters	1.1–1.4	1.3–4.5	45–85

Table 25.2 Properties, and specific properties, of composites

Material	Density ρ (Mg m^{-3})	Young's modulus E(GPa)	Strength σ_y(MPa)	Fracture toughness K_{IC}(MPa m$^{1/2}$)	E/ρ	E$^{1/2}$/ρ	E$^{1/3}$/ρ	σ_y/ρ
Composites								
CFRP, 58% uniaxial C in epoxy	1.5	189	1050	32−45	126	9	3.8	700
GFRP, 50% uniaxial glass in polyester	2.0	48	1240	42−60	24	3.5	1.8	620
Kevlar-epoxy (KFRP), 60% uniaxial Kevlar in epoxy	1.4	76	1240	−	54	6.2	3.0	886
Metals								
High-strength steel	7.8	207	1000	100	27	1.8	0.76	128
Aluminium alloy	2.8	71	500	28	25	3.0	1.5	179

Fig. 25.1. (**a**) When loaded along the fibre direction the fibres and matrix of a continuous-fibre composite suffer equal strains. (**b**) When loaded across the fibre direction, the fibres and matrix see roughly equal stress; particulate composites are the same. (**c**) A 0–90° laminate has high and low modulus directions; a 0–45–90–135° laminate is nearly isotropic.

Modulus

When two linear-elastic materials (though with different moduli) are mixed, the mixture is also linear-elastic. The modulus of a fibrous composite when loaded *along* the fibre direction (Fig. 25.1a) is a linear combination of that of the fibres, E_f, and the matrix, E_m

$$E_{c\parallel} = V_f E_f + (1 - V_f)E_m \tag{25.1}$$

where V_f is the volume fraction of fibres (see Book 1, Chapter 6). The modulus of the same material, loaded across the fibres (Fig. 25.1b) is much less – it is only

$$E_{c\perp} = \left\{ \frac{V_f}{E_f} + \frac{1 - V_f}{E_m} \right\}^{-1} \tag{25.2}$$

(see Book 1, Chapter 6 again).

Table 25.1 gives E_f and E_m for common composites. The moduli $E_\parallel$ and $E_\perp$ for a composite with, say, 50% of fibres, differ greatly: a uniaxial composite (one in which all the fibres are aligned in one direction) is exceedingly anisotropic. By using a cross-weave of fibres (Fig. 25.1c) the moduli in the 0 and 90° directions can be made equal, but those at 45° are still very low. Approximate isotropy can be restored by laminating sheets, rotated through 45°, to give a plywood-like *fibre laminate*.
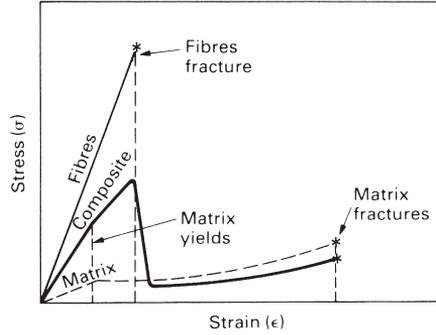
Fig. 25.2. The stress–strain curve of a continuous fibre composite (heavy line), showing how it relates to those of the fibres and the matrix (thin lines). At the peak the fibres are on the point of failing.

Tensile strength and the critical fibre length

Many fibrous composites are made of strong, brittle fibres in a more ductile polymeric matrix. Then the stress–strain curve looks like the heavy line in Fig. 25.2. The figure largely explains itself. The stress–strain curve is linear, with slope E (eqn. 25.1) until the matrix yields. From there on, most of the extra load is carried by the fibres which continue to stretch elastically until they fracture. When they do, the stress drops to the yield strength of the matrix (though not as sharply as the figure shows because the fibres do not all break at once). When the matrix fractures, the composite fails completely.

In any structural application it is the peak stress which matters. At the peak, the fibres are just on the point of breaking and the matrix has yielded, so the stress is given by the *yield* strength of the matrix, σ_y^m, and the *fracture* strength of the fibres, σ_f^f, combined using a rule of mixtures

$$\sigma_{TS} = V_f \sigma_f^f + (1 - V_f)\sigma_y^m. \tag{25.3}$$

This is shown as the line rising to the right in Fig. 25.3. Once the fibres have fractured, the strength rises to a second maximum determined by the fracture strength of the matrix

$$\sigma_{TS} = (1 - V_f)\sigma_f^m \tag{25.4}$$

where σ_f^m is the fracture strength of the matrix; it is shown as the line falling to the right on Fig. 25.3. The figure shows that adding too few fibres does more harm than good: a critical volume fraction $V_{f_{crit}}$ of fibres must be exceeded to give an increase in strength. If there are too few, they fracture before the peak is reached and the ultimate strength of the material is reduced.

For many applications (e.g. body pressings), it is inconvenient to use continuous fibres. It is a remarkable feature of these materials that chopped fibre composites (convenient for moulding operations) are nearly as strong as those with continuous fibres, provided the fibre length exceeds a critical value.

Consider the peak stress that can be carried by a chopped-fibre composite which has a matrix with a yield strength in shear of $\sigma_s^m (\sigma_s^m \approx \frac{1}{2}\sigma_y^m)$. Figure 25.4 shows that the axial force transmitted to a fibre of diameter d over a little segment δx of its length is

$$\delta F = \pi d \sigma_s^m \delta x. \tag{25.5}$$
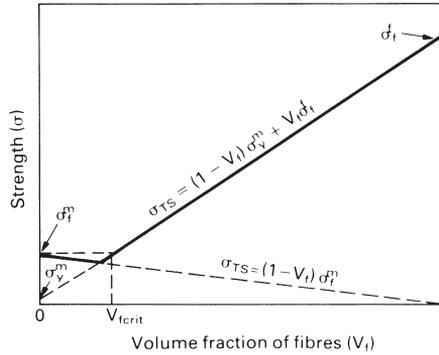
Fig. 25.3. The variation of peak stress with volume fraction of fibres. A minimum volume fraction ($V_{f_{crit}}$) is needed to give any strengthening.
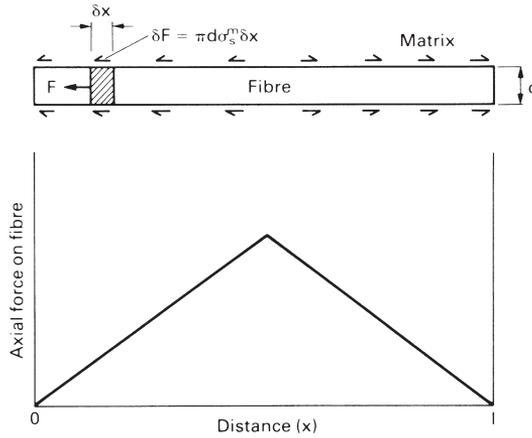
Fig. 25.4. Load transfer from the matrix to the fibre causes the tensile stress in the fibre to rise to peak in the middle. If the peak exceeds the fracture strength of the fibre, it breaks.

The force on the fibre thus increases from zero at its end to the value

$$F = \int_0^x \pi d\sigma_s^m \mathrm{d}x = \pi d\sigma_s^m x \tag{25.6}$$

at a distance x from the end. The force which will just break the fibre is

$$F_c = \frac{\pi d^2}{4}\sigma_f^f. \tag{25.7}$$

Equating these two forces, we find that the fibre will break at a distance

$$x_c = \frac{d}{4}\frac{\sigma_f^f}{\sigma_s^m} \tag{25.8}$$

from its end. If the fibre length is less than $2x_c$, the fibres do not break – but nor do they carry as much load as they could. If they are much longer than $2x_c$, then nothing is gained by the extra length. The optimum strength (and the most effective use of the
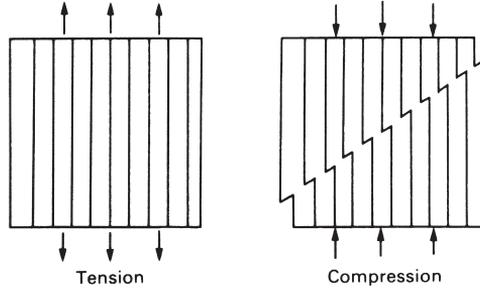
Fig. 25.5. Composites fail in compression by kinking, at a load which is lower than that for failure in tension.

fibres) is obtained by chopping them to the length $2x_c$ in the first place. The average stress carried by a fibre is then simply $\sigma_f^f/2$ and the peak strength (by the argument developed earlier) is

$$\sigma_{\text{TS}} = \frac{V_f \sigma_f^f}{2} + (1 - V_f)\sigma_y^m. \tag{25.9}$$

This is more than one-half of the strength of the continuous-fibre material (eqn. 25.3). Or it is if all the fibres are aligned along the loading direction. That, of course, will not be true in a chopped-fibre composite. In a car body, for instance, the fibres are randomly oriented in the plane of the panel. Then only a fraction of them – about $\frac{1}{4}$ – are aligned so that much tensile force is transferred to them, and the contributions of the fibres to the stiffness and strength are correspondingly reduced.

The compressive strength of composites is less than that in tension. This is because the fibres buckle or, more precisely, they *kink* – a sort of co-operative buckling, shown in Fig. 25.5. So while brittle ceramics are best in compression, composites are best in tension.

Toughness

The *toughness* G_c of a composite (like that of any other material) is a measure of the energy absorbed per unit crack area. If the crack simply propagated straight through the matrix (toughness G_c^m) and fibres (toughness G_c^f), we might expect a simple rule-of-mixtures

$$G_c = V_f G_c^f + (1 - V_f)G_c^m. \tag{25.10}$$

But it does not usually do this. We have already seen that, if the length of the fibres is less than $2x_c$, they will not fracture. And if they do not fracture they must instead pull out as the crack opens (Fig. 25.6). This gives a major new contribution to the toughness. If the matrix shear strength is σ_s^m (as before), then the work done in pulling a fibre out of the fracture surface is given approximately by

$$\int_0^{l/2} F dx = \int_0^{l/2} \pi d\sigma_s^m x dx = \pi d\sigma_s^m \frac{l^2}{8} \tag{25.11}$$

The number of fibres per unit crack area is $4V_f/\pi d^2$ (because the volume fraction is the same as the *area* fraction on a plane perpendicular to the fibres). So the total work done per unit crack area is
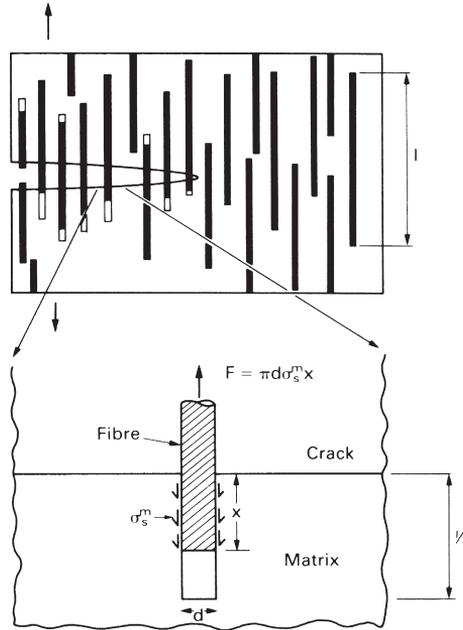
Fig. 25.6. Fibres toughen by pulling out of the fracture surface, absorbing energy as the crack opens.

$$G_c = \pi d \sigma_s^m \frac{l^2}{8} \times \frac{4V_f}{\pi d^2} = \frac{V_f}{2d} \sigma_s^m l^2. \tag{25.12}$$

This assumes that l is less than the critical length $2x_c$. If l is greater than $2x_c$ the fibres will *not* pull out, but will break instead. Thus optimum toughness is given by setting $l = 2x_c$ in eqn. (25.12) to give
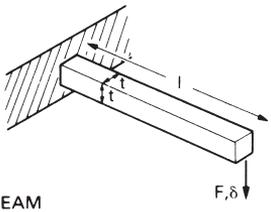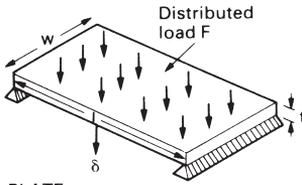
$$G_c = \frac{2V_f}{d} \sigma_s^m x_c^2 = \frac{2V_f}{d} \sigma_s^m \left(\frac{d}{4} \frac{\sigma_f^f}{\sigma_s^m} \right)^2 = \frac{V_f d}{8} \frac{(\sigma_f^f)^2}{\sigma_s^m}. \tag{25.13}$$

The equation says that, to get a high toughness, you should use strong fibres in a weak matrix (though of course a weak matrix gives a low strength). This mechanism gives CFRP and GFRP a toughness (50 kJ m^{-2}) far higher than that of either the matrix (5 kJ m^{-2}) or the fibres (0.1 kJ m^{-2}); without it neither would be useful as an engineering material.

Applications of composites

In designing transportation systems, weight is as important as strength. Figure 25.7 shows that, depending on the geometry of loading, the component which gives the least *deflection* for a given weight is that made of a material with a maximum E/ρ (ties in tension), $E^{1/2}/\rho$ (beam in bending) or $E^{1/3}/\rho$ (plate in bending).

When E/ρ is the important parameter, there is nothing to choose between steel, aluminium or fibre glass (Table 25.2). But when $E^{1/2}/\rho$ is controlling, aluminium is better than steel: that is why it is the principal airframe material. Fibreglass is not

Mode of loading	Optimum stiffness	Optimum strength
TIE	$\delta = \dfrac{Fl}{Et^2}$ $M = \rho lt^2$ $\quad = \left(\dfrac{l^2 F}{\delta}\right)\dfrac{\rho}{E}$ Maximise $\dfrac{E}{\rho}$	$\sigma_y = \dfrac{F}{t^2}$ $M = \rho lt^2$ $\quad = Fl\left(\dfrac{\rho}{\sigma_y}\right)$ Maximise $\dfrac{\sigma_y}{\rho}$
BEAM	$\delta = \dfrac{4Fl^3}{Et^4}$ $M = \rho lt^2$ $\quad = 2\left(\dfrac{Fl^5}{\delta}\right)^{1/2}\left(\dfrac{\rho}{E^{1/2}}\right)$ Maximise $\dfrac{E^{1/2}}{\rho}$	$\sigma_y = \dfrac{6Fl}{t^3}$ $M = \rho lt^2$ $\quad = l(6Fl)^{2/3}\left(\dfrac{\rho}{\sigma_y^{2/3}}\right)$ Maximise $\dfrac{\sigma_y^{2/3}}{\rho}$
PLATE	$\delta = \dfrac{5Fl^3}{32Ewt^3}$ $M = \rho\, lwt$ $\quad = l^2\left(\dfrac{5Fw^2}{32\delta}\right)^{1/3}\left(\dfrac{\rho}{E^{1/3}}\right)$ Maximise $\dfrac{E^{1/3}}{\rho}$	$\sigma_y = \dfrac{3Fl}{4\,wt^2}$ $M = \rho\, lwt$ $\quad = \left(\dfrac{3Fl^3 w}{4}\right)^{1/2}\left(\dfrac{\rho}{\sigma_y^{1/2}}\right)$ Maximise $\dfrac{\sigma_y^{1/2}}{\rho}$

E = Young's modulus; σ_y = Yield strength; ρ = Density

Fig. 25.7. The combination of properties which maximise the stiffness-to-weight ratio and the strength-to-weight ratio, for various loading geometries.

significantly better. Only CFRP and KFRP offer a real advantage, and one that is now exploited extensively in aircraft structures. This advantage persists when $E^{1/3}/\rho$ is the determining quantity – and for this reason both CFRP and KFRP find particular application in floor panels and large load-bearing surfaces like flaps and tail planes.

In some applications it is strength, not stiffness, that matters. Figure 25.7 shows that the component with the greatest *strength* for a given weight is that made of the material with a maximum σ_y/ρ (ties in tension), $\sigma_y^{2/3}/\rho$ (beams in bending) or $\sigma_y^{1/2}/\rho$ (plates in bending). Even when σ_y/ρ is the important parameter, composites are better than metals (Table 25.2), and the advantage grows when $\sigma_y^{2/3}/\rho$ or $\sigma_y^{1/2}/\rho$ are dominant.

Despite the high cost of composites, the weight-saving they permit is so great that their use in trains, trucks and even cars is now extensive. But, as this chapter illustrates, the engineer needs to understand the material and the way it will be loaded in order to use composites effectively.

Particulate composites

Particulate composites are made by blending silica flour, glass beads, even sand into a polymer during processing.

Particulate composites are much less efficient in the way the filler contributes to the strength. There is a small gain in stiffness, and sometimes in strength and toughness, but it is far less than in a fibrous composite. Their attraction lies more in their low cost and in the good wear resistance that a hard filler can give. Road surfaces are a good example: they are either *macadam* (a particulate composite of gravel in bitumen, a polymer) or *concrete* (a composite of gravel in cement, for which see Chapter 20).

Cellular solids, or foams

Many natural materials are cellular: wood and bone, for example; cork and coral, for instance. There are good reasons for this: cellular materials permit an optimisation of stiffness, or strength, or of energy absorption, for a given weight of material. These natural foams are widely used by people (wood for structures, cork for thermal insulation), and synthetic foams are common too: cushions, padding, packaging, insulation, are all functions filled by cellular polymers. Foams give a way of making solids which are very light and, if combined with stiff skins to make sandwich panels, they give structures which are exceptionally stiff and light. The engineering potential of foams is considerable, and, at present, incompletely realised.

Most polymers can be foamed easily. It can be done by simple mechanical stirring or by blowing a gas under pressure into the molten polymer. But by far the most useful method is to mix a chemical blowing agent with the granules of polymer before processing: it releases CO_2 during the heating cycle, generating gas bubbles in the final moulding. Similar agents can be blended into thermosets so that gas is released during curing, expanding the polymer into a foam; if it is contained in a closed mould it takes up the mould shape accurately and with a smooth, dense, surface.

The properties of a foam are determined by the properties of the polymer, and by the *relative density*, ρ/ρ_s: the density of the foam (ρ) divided by that of the solid (ρ_s) of which it is made. This plays the role of the volume fraction V_f of fibres in a composite, and all the equations for foam properties contain ρ/ρ_s. It can vary widely, from 0.5 for a dense foam to 0.005 for a particularly light one.

The cells in foams are polyhedral, like grains in a metal (Fig. 25.8). The cell walls, where the solid is concentrated, can be open (like a sponge) or closed (like a flotation foam), and they can be equiaxed (like the polymer foam in the figure) or elongated
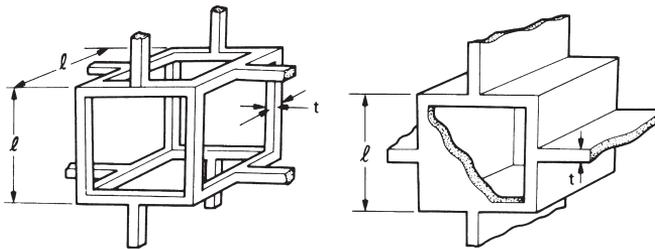
Fig. 25.8. Polymeric foams, showing the polyhedral cells. Some foams have closed cells, others have cells which are open.
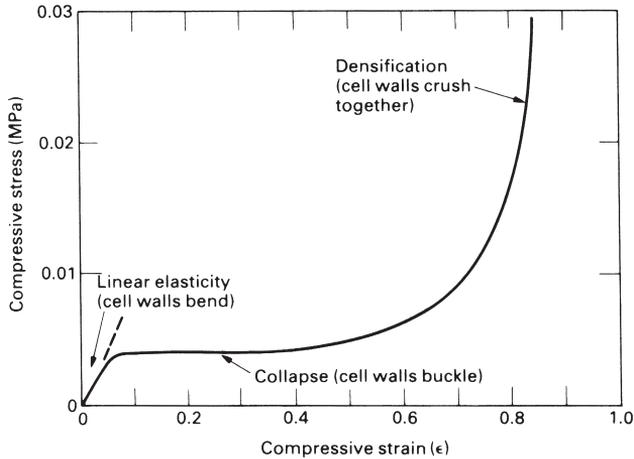
Fig. 25.9. The compressive stress–strain curve for a polymeric foam. Very large compressive strains are possible, so the foam absorbs a lot of energy when it is crushed.

(like cells in wood). But the aspect of structures which is most important in determining properties is none of these; it is the relative density. We now examine how foam properties depend on ρ/ρ_s and on the properties of the polymer of which it is made (which we covered in Chapter 23).

Mechanical properties of foams

When a foam is compressed, the stress–strain curve shows three regions (Fig. 25.9). At small strains the foam deforms in a *linear-elastic* way: there is then a *plateau* of deformation at almost constant stress; and finally there is a region of *densification* as the cell walls crush together.

At small strains the cell walls at first *bend*, like little beams of modulus E_s, built in at both ends. Figure 25.10 shows how a hexagonal array of cells is distorted by this bending. The deflection can be calculated from simple beam theory. From this we obtain the stiffness of a unit cell, and thus the modulus E of the foam, in terms of the length l and thickness t of the cell walls. But these are directly related to the relative density: $\rho/\rho_s = (t/l)^2$ for open-cell foams, the commonest kind. Using this gives the foam modulus as

$$E = E_s \left(\frac{\rho}{\rho_s} \right)^2 . \tag{25.14}$$

Real foams are well described by this formula. Note that foaming offers a vast range of modulus: ρ/ρ_s can be varied from 0.5 to 0.005, a factor of 10^2, by processing, allowing E to be varied over a factor of 10^4.

Linear-elasticity, of course, is limited to small strains (5% or less). Elastomeric foams can be compressed far more than this. The deformation is still recoverable (and thus elastic) but is non-linear, giving the plateau on Fig. 25.9. It is caused by the *elastic*
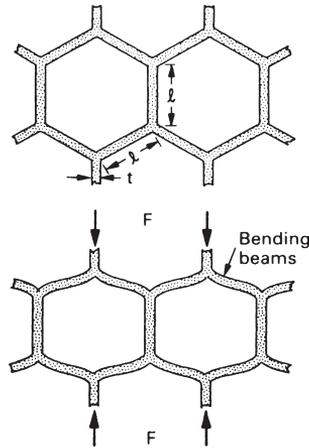
Fig. 25.10. Cell wall bending gives the linear-elastic portion of the stress–strain curve.
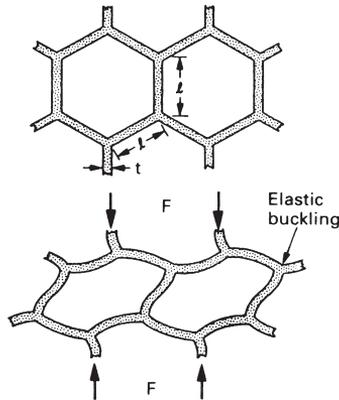

Fig. 25.11. When an elastomeric foam is compressed beyond the linear region, the cell walls buckle elastically, giving the long plateau shown in Fig. 25.9.

buckling of the columns or plates which make up the cell edges or walls, as shown in Fig. 25.11. Again using standard results of beam theory, the *elastic collapse stress* σ_{el}^* can be calculated in terms of the density ρ. The result is

$$\sigma_{el}^* = 0.05E_s\left(\frac{\rho}{\rho_s}\right)^2. \tag{25.15}$$

As before, the strength of the foam is controlled by the density, and can be varied at will through a wide range. Low-density ($\rho/\rho_s = 0.01$) elastomeric foams collapse under tiny stresses; they are used to package small, delicate instruments. Denser foams ($\rho/\rho_s = 0.05$) are used for seating and beds: their moduli and collapse strengths are 25 times larger. Still denser foams are used for packing heavier equipment: appliances or small machine tools, for instance.
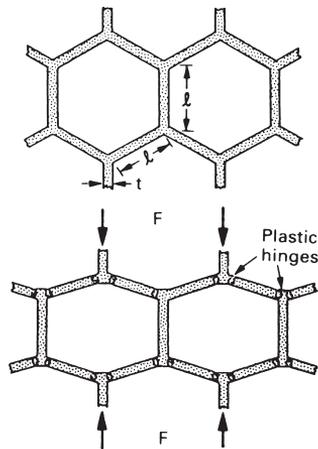
Fig. 25.12. When a plastic foam is compressed beyond the linear region, the cell walls bend plastically, giving a long plateau exactly like that of Fig. 25.9.

Cellular materials can collapse by another mechanism. If the cell-wall material is *plastic* (as many polymers are) then the foam as a whole shows plastic behaviour. The stress–strain curve still looks like Fig. 25.9, but now the plateau is caused by plastic collapse. Plastic collapse occurs when the moment exerted on the cell walls exceeds its fully plastic moment, creating plastic hinges as shown in Fig. 25.12. Then the collapse stress σ_{pl}^* of the foam is related to the yield strength σ_y of the wall by

$$\sigma_{pl}^* = 0.3\sigma_y\left(\frac{\rho}{\rho_s}\right)^{3/2}. \tag{25.16}$$

Plastic foams are good for the kind of packaging which is meant to absorb the energy of a single impact: polyurethane automobile crash padding, polystyrene foam to protect a television set if it is accidentally dropped during delivery. The long plateau of the stress–strain curve absorbs energy but the foam is damaged in the process.

Materials that can be engineered

The materials described in this chapter differ from most others available to the designer in that their properties can be engineered to suit, as nearly as possible, the application. The stiffness, strength and toughness of a composite are, of course, controlled by the type and volume fraction of fibres. But the materials engineering can go further than this, by orienting or laminating the fibre weave to give directional properties, or to reinforce holes or fixing points, or to give a stiffness which varies in a controlled way across a component. Foaming, too, allows new degrees of freedom to the designer. Not only can the stiffness and strength be controlled over a vast range (10^4 or more) by proper choice of matrix polymer and foam density, but gradients of foam density and thus of properties can be designed-in. Because of this direct control

over properties, both sorts of composites offer special opportunities for designing *weight-optimal structures*, particularly attractive in aerospace and transport. Examples of this sort of design can be found in the books listed below.

Further reading

M. R. Piggott, *Load Bearing Fibre Composites*, Pergamon Press, 1980.
A. F. Johnson, *Engineering Design Properties of GFRP*, British Plastics Federation, 1974.
M. Grayson (editor), *Encyclopedia of Composite Materials and Components*, Wiley, 1983.
P. C. Powell and A. J. Ingen Housz, *Engineering with Polymers*, 2nd edition, Chapman and Hall, 1998.
L. J. Gibson and M. F. Ashby, *Cellular Solids*, 2nd edition, Butterworth-Heinemann, 1997.
D. Hull, *An Introduction to Composite Materials*, Cambridge University Press, 1981.
N. C. Hillyard (editor), *Mechanics of Cellular Plastics*, Applied Science Publishers, 1982.

Problems

25.1 A unidirectional fibre composite consists of 60% by volume of Kevlar fibres in a matrix of epoxy. Find the moduli $E_{c\|}$ and $E_{c\perp}$. Comment on the accuracy of your value for $E_{c\perp}$. Use the moduli given in Table 25.1, and use an average value where a range of moduli is given.

Answers: 77 GPa and 9 GPa.

25.2 A unidirectional fibre composite consists of 60% by volume of continuous type-1 carbon fibres in a matrix of epoxy. Find the maximum tensile strength of the composite. You may assume that the matrix yields in tension at a stress of 40 MPa.

Answer: 1336 MPa.

25.3 A composite material for a car-repair kit consists of a random mixture of short glass fibres in a polyester matrix. Estimate the maximum toughness G_c of the composite. You may assume that: the volume fraction of glass is 30%; the fibre diameter is 15 μm; the fracture strength of the fibres is 1400 MPa; and the shear strength of the matrix is 30 MPa.

Answer: 37 kJ m^{-2}.

25.4 Calculate the critical length $2X_c$ of the fibres in Problem 25.3. How would you expect G_c to change if the fibres were substantially longer than $2X_c$?

Answer: 0.35 mm.